Example: consider the set of "Tall Persons among 9 students". One possibility for the membership function could be:

$$\Lambda(x)=\begin{cases}0 & x\le 150\,Cm\\ \dfrac{(x-150)}{40} & 150\,Cm<x\le 190\,Cm\\ \dfrac{(230-x)}{40} & 190\,Cm<x\le 230\,Cm\\ 0 & x>230\,Cm\end{cases}$$

and the set will be stated as follows:

$$\tilde{T}=\{(x,\Lambda(x))\,|\,where\;x\;is\;the\;height\;of\;the\;student\;in\;Cm,\Lambda(x)\;is\;his\;\deg ree\;of\;membership\}$$

$$=\{(150,0),(159,0.225),(168,0.45),(177,0.675),(190,1),(203,0.675),(212,0.45),(221,0.225),(230,0)\}$$

1. $\text{supp}(\tilde{T})=\{159,168,177,190,203,212,221\}$
2. $hgt(\tilde{T})=\sup_x \Lambda_{\tilde{T}}(x)=1$.
3. to find the complement of the fuzzy set you can use the definition directly and you can find the more general case which is to find the complement membership function $\Phi_{\tilde{T}^c}(x)=1-\Lambda_{\tilde{T}}(x)$:

$$\Phi_{\tilde{T}^c}(x)=1-\Lambda_{\tilde{T}}(x)$$

$$=1-\begin{cases}0 & x\le 150\,Cm\\ \dfrac{(x-150)}{40} & 150\,Cm<x\le 190\,Cm\\ \dfrac{(230-x)}{40} & 190\,Cm<x\le 230\,Cm\\ 0 & x>230\,Cm\end{cases}=\begin{cases}1 & x\le 150\,Cm\\ 1-\dfrac{(x-150)}{40} & 150\,Cm<x\le 190\,Cm\\ 1-\dfrac{(230-x)}{40} & 190\,Cm<x\le 230\,Cm\\ 1 & x>230\,Cm\end{cases}$$

$$\tilde{T}^c=\{(150,1),(159,0.775),(168,0.55),(177,0.325),(190,0),(203,0.325),(212,0.55),(221,0.775),(230,1)\}$$

4. another choice for the membership function is :

$$\Omega(x)=\begin{cases}0 & x\le 160\,Cm\\ \dfrac{(x-160)}{30} & 160\,Cm<x\le 190\,Cm\\ \dfrac{(220-x)}{30} & 190\,Cm<x\le 220\,Cm\\ 0 & x>220\,Cm\end{cases}$$

$$\tilde{S}=\{(x,\Omega(x))\,|\,where\;x\;is\;the\;height\;of\;the\;student\;in\;Cm,\Omega(x)\;is\;his\;\deg ree\;of\;membership\}$$

$$=\{(150,0),(159,0),(168,0.26),(177,0.56),(190,1),(203,0.56),(212,0.26),(221,0),(230,0)\}$$

5. $\tilde{S} \subset \tilde{T}$.

6. $T\tilde{A} = \{(168, 0.45), (177, 0.675), (190, 1), (203, 0.675), (212, 0.45)\}$, $T\tilde{A} \subseteq \tilde{T}$.

7. $T\tilde{B} = \{(177, 0.675), (190, 1), (203, 0.675)\}$, $T\tilde{B} \subseteq T\tilde{A}$, $T\tilde{A} \subseteq \tilde{T}$, $T\tilde{B} \subseteq \tilde{T}$.

8. $S\tilde{A} = \{(168, 0.26), (177, 0.56), (190, 1), (203, 0.56), (212, 0.26)\}$, $S\tilde{A} \subseteq \tilde{S}$, $S\tilde{A} \subset \tilde{T}$.

9. $S\tilde{B} = \{(177, 0.56), (190, 1), (212, 0.26)\}$, $S\tilde{B} \subseteq S\tilde{A}$, $S\tilde{B} \subset T\tilde{A}$, $S\tilde{B} \subseteq \tilde{S}$, $S\tilde{B} \subset \tilde{T}$.

10. $T\tilde{A} \cup S\tilde{A} =$

$$= \{(168, \max(0.45, 0.26)), (177, \max(0.675, 0.56)), (190, \max(1,1)), (203, \max(0.675, 0.56)), (212, \max(0.45, 0.26))\}.$$

$$= \{(168, 0.45), (177, 0.675), (190, 1), (203, 0.675), (212, 0.45)\}.$$

$T\tilde{B} \cup S\tilde{B} =$

$$= \{(177, \max(0.675, 0.56)), (190, \max(1,1)), (203, \max(0.675, 0)), (212, \max(0.26, 0))\}.$$

$$= \{(177, 0.675), (190, 1), (203, 0.675), (212, 0.26)\}.$$

11. $T\tilde{A} \cap S\tilde{A} =$

$$= \{(168, \min(0.45, 0.26)), (177, \min(0.675, 0.56)), (190, \min(1,1)), (203, \min(0.675, 0.56)), (212, \min(0.45, 0.26))\}.$$

$$= \{(168, 0.26), (177, 0.56), (190, 1), (203, 0.56), (212, 0.26)\}.$$

$T\tilde{B} \cap S\tilde{B} =$

$$= \{(177, \min(0.675, 0.56)), (190, \min(1,1)), (203, \min(0.675, 0)), (212, \min(0.26, 0))\}.$$

$$= \{(177, 0.56), (190, 1), (203, 0), (212, 0)\}.$$

$$= \{(177, 0.56), (190, 1)\}.$$

12. now we wish to find the α-Cuts A_1, $A_{0.8}$, $A_{0.6}$, $A_{0.5}$, $A_{0.3}$, $A_{0.2}$ of $\tilde{T}$:

$$A_{0.2} = \{159, 168, 177, 190, 203, 212, 221\}.$$

$$A_{0.3} = \{168, 177, 190, 203, 212\}.$$

$$A_{0.5} = \{177, 190, 203\}.$$

$$A_{0.6} = \{177, 190, 203\}.$$

$$A_{0.8} = \{190\}.$$

$$A_1 = \{190\}.$$

LINGUISTIC VARIABLES [Witold, 2007]

One can often deal with variables describing phenomena of physical or human systems assuming a finite, quite small number of descriptors. We often describe observations about a phenomenon by characterizing its states that we naturally translate in terms of the idea of a variable.

For instance, we may refer to an environment through words such as comfortable, sunny, and neat. In particular, we can qualify the environment condition through the variable temperature with values chosen in a range such as the interval X = [0, 40]. Alternatively, temperature could be qualified using labels such as cold, comfortable, and warm. A precise numerical value such as 20°C seems simpler to characterize the environment than the ill-defined term comfortable. But the linguistic label comfortable is a choice of one out of three values, whereas 20°C is a choice out of many.

The statement could be strengthened if the underlying meaning of comfortable is conceived as about 20°C. Although the numerical quantity 20°C can be visualized as a point in a set, the linguistic temperature value *comfortable* can be viewed as a collection of temperature values in a bounded region centered in 20°C.

In these circumstances, fuzzy sets provide a way to map a finite term set to a linguistic scale whose values are fuzzy sets. In general, it is difficult to find incontestable thresholds, such as 15°C and 30°C for instance, which allows us to assign *Cold* = [0,15], *Comfortable* = [15, 30], and *Warm* = [30, 40]. *Cold*, *comfortable*, and *warm* are fuzzy sets instead of single numbers or sets (intervals).

As fuzzy sets concern the representation of collections with unclear boundaries by means of membership functions taking values in an ordered set of membership values, they provide a means to interface numerical and linguistic quantities, a way to link computing with words and granular computing.

In contrast to the idea of numeric variables as being commonly used, the notion of linguistic variable can be regarded as a variable whose values are fuzzy sets. In general, linguistic variables may assume values consisting of words or sentences expressed in a certain language. Formally, a linguistic variable is characterized by a quintuple $\langle X, T(X), G, M \rangle$ where X is the name of the variable, T(x) is a term set of X whose elements are labels L of linguistic values of X, G a grammar that generates the labels of X, M a semantic rule that assigns to each label $L \in T(X)$ a meaning whose realization is a fuzzy set on the universe X whose base variable is *x*.

Example [Witold, 2007].

Let us consider the linguistic variable of temperature. Here, the linguistic variable is formalized by explicitly identifying all the components of the formal definition:

X = temperature,

X = [0, 40]

T(temperature) = {*cold*, *comfortable*, *warm*}

M(*cold*) = C, M(*comfortable*) = F and M(*warm*) = W,

where C, F, and W are fuzzy sets whose membership functions are *C(x), F(x), and W(x)*.

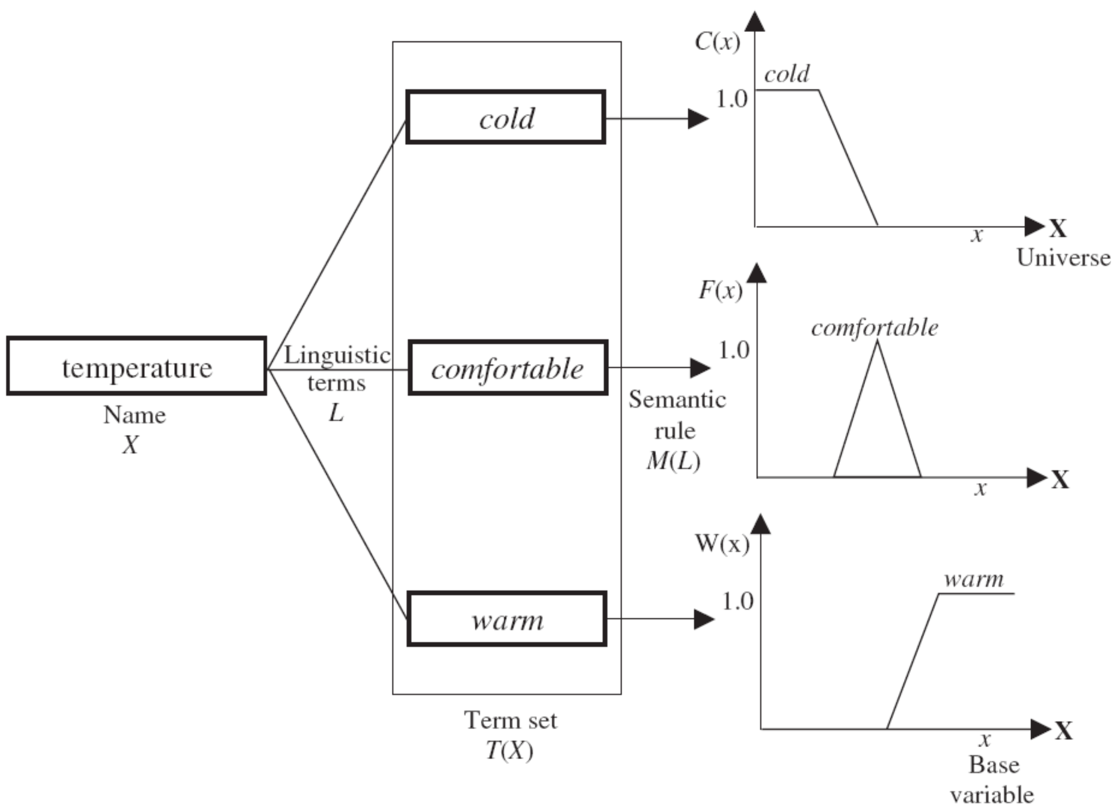

Fig.(12): fuzzy variable temperature

Hedges

Hedges are linguistic modifiers on membership functions; they are mappings that transform a membership function into another. The hedges are important feature of fuzzy systems; these operations are provided in an effort to maintain close ties to natural language, and to allow for the generation of fuzzy statements through mathematical calculations. For example, Consider the membership function Tall(x) for tall persons, a possible option for the following possible hedges would be:

– *Not* Tall is 1-Tall(x)
– *Less* Tall is $Tall(x)^{0.5}$.
– *Little* Tall is $Tall(x)^{1.3}$.
– *Slightly* Tall is $Tall(x)^{1.7}$.
– *Very* Tall is $Tall(x)^{2}$.
– *Extremely* Tall is $Tall(x)^{3}$.

Linguistic Rules

Rule-based models play a central role in fuzzy modeling. Fuzzy rules capture relationships among fuzzy variables and provide a mechanism to link linguistic descriptions of systems with their computational realizations.

In their generic format, rules forming a core of rule-based systems come in the form of conditional statements.

If (input variable is A) then (output variable is B) eq.(18)

where A and B standing in the ''If'' and ''then'' parts of the rules are descriptors of some pieces of knowledge about the domain (problem to be represented), the part (input variable is A) is called the antecedents, where (output variable is B) is the consequent. The rule itself expresses a certain relationship between these input and output variables.

For instance, the rule ''if the temperature is high then the electricity demand is high'' captures a piece of domain knowledge that is essential to some specific planning activities exercised by an electric company.

Both antecedents and consequents can involve several linguistic variables. If this is the case, the system is called a *multi-input-multi-output* (*MIMO*) fuzzy system, Such systems have several insignals and outsignals:

$$\text{If } \tilde{u}_1 \text{ is } \tilde{A}_1^j \text{ and } \tilde{u}_2 \text{ is } \tilde{A}_2^k \text{ and},\ldots,\text{and } \tilde{u}_n \text{ is } \tilde{A}_n^L \text{ Then } \tilde{y}_1 \textit{ is } \tilde{B}_1^p \textit{ and } \tilde{y}_2 \textit{ is } \tilde{B}_2^q \text{ and},\ldots,\text{and } \tilde{y}_m \textit{ is } \tilde{B}_m^s \quad \text{eq.(19)}$$

Also *multi-input-single-output* (*MISO*) systems, with several insignals but only one outsignal:

$$\text{If } \tilde{u}_1 \text{ is } \tilde{A}_1^j \text{ and } \tilde{u}_2 \text{ is } \tilde{A}_2^k \text{ and},\ldots,\text{ and } \tilde{u}_n \text{ is } \tilde{A}_n^L \text{ Then } \tilde{y} \textit{ is } \tilde{B}^p \qquad \text{eq.(20)}$$

where j, k, L, m, n, p, q, s $\in$ N .

Example.

The following shows an example of fuzziness used for speed control. As insignals we have the actual speed *v* (km/h) and the load *l* (N) of the car. Load components are e.g. force *F* and friction $F\mu$.

As linguistic variables we have {speed, load}, for speed we use the linguistic values {*LS* (low speed), *NS* (normal speed), *HS* (high speed)}, and for load {*LL* (low load), *NL* (normal load), *HL* (high load)}.

A Bell-shaped membership functions can be a good choice and as midpoint in *NS* we use 70 km/h which also is the constant speed we try to maintain. The choice of precise shape and transposition of membership functions is left open at this point. Low load appear e.g. downhill and high load uphill.

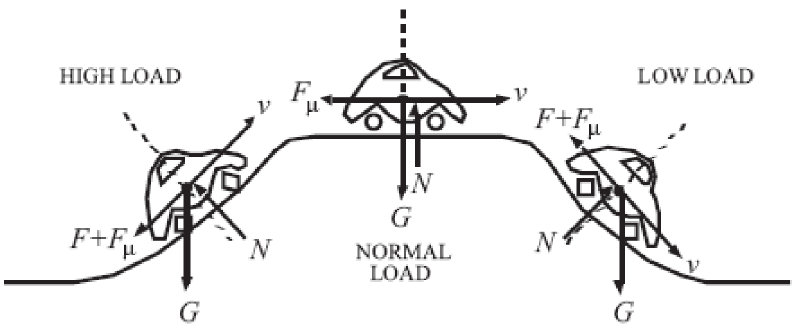

Fig.(13): Physical Sketch for the car speed control problem

The rule base for speed control could be as follows.

$R1$: IF v is LS AND l is NL THEN v is NS

$R2$: IF v is LS AND l is HL THEN v is HS

$R3$: IF v is NS AND l is LL THEN v is LS

$R4$: IF v is NS AND l is HL THEN v is HS

$R5$: IF v is HS AND l is LL THEN v is LS

$R6$: IF v is HS AND l is NL THEN v is NS