

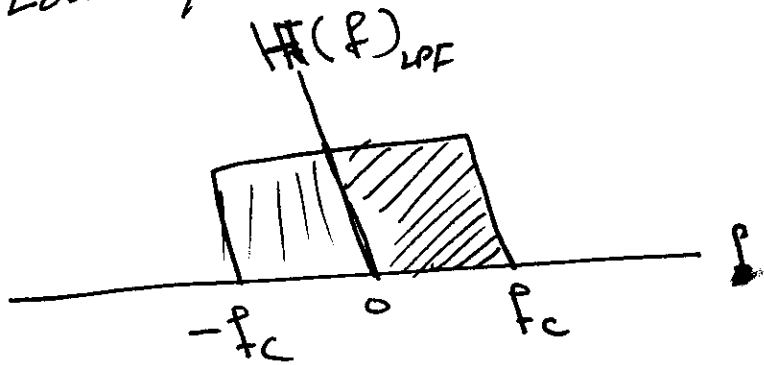
6- Filter type

Filter it's electrical element used to select some type of frequencies than the other. There are Four type of filter

1- Low pass filter (LPF)

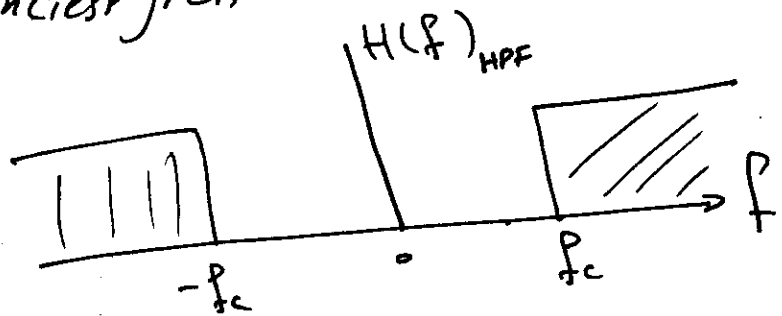
This filter Pass only Low frequencies as shown Less than cut-off frequency

LPF



2- High Pass filter (HPF)

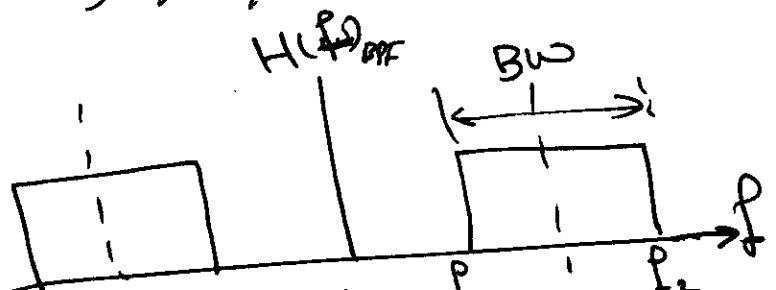
This filter Pass frequencies greater than cut off freq. f_c



3- Band Pass Filter (BPF)

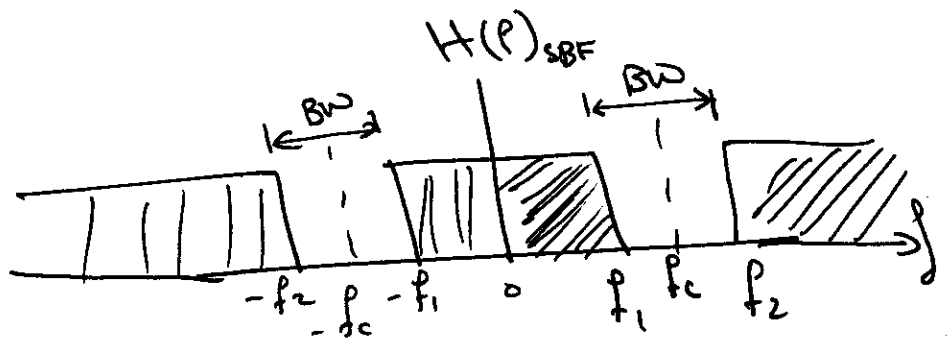
This filter select Band of frequencies between f_1 & f_2

$$BW = f_2 - f_1$$



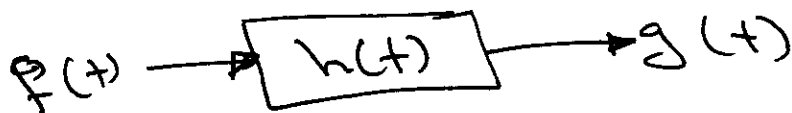
4- Stop Band Filter (SBF)

This filter reject only Band of frequencies than the other



Convolution

The output of any system have by convoluted
Input with system Impulse response



$$g(t) = f(t) * h(t)$$

--- (12)

where

$$g(t) = \int_{-\infty}^{\infty} f(\tau) \cdot h(t-\tau) d\tau$$

$f(t)$: Input wave form

$h(t)$: System Impulse response

$g(t)$: output wave form

The output frequency domain done by mult
Signal frequency ~~response~~ ^{Domain} by system freq. response

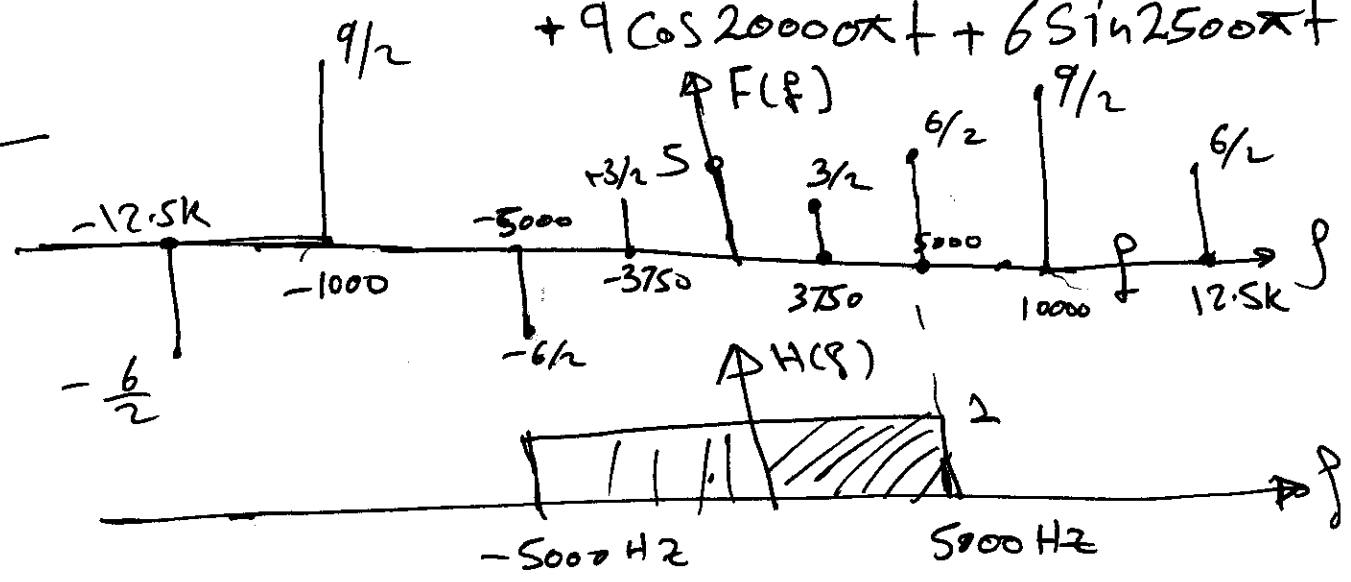
$$G(\omega) = F(\omega) * H(\omega) \quad \text{--- (13)}$$

For that the output of any filter done by multiply signal $F(\omega)$ spectrum by filter spectrum.

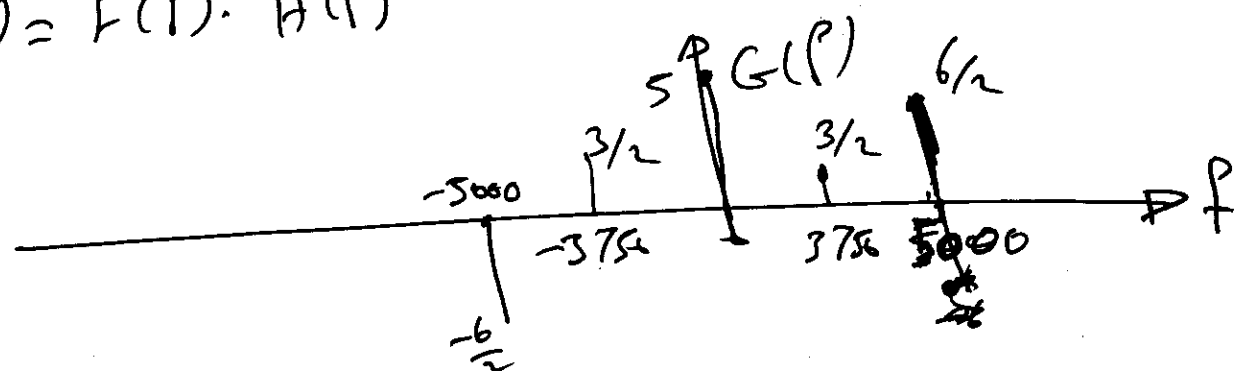
Ex: An signal $f(t)$ Input ~~pass~~ through LPF of frequency 5000 Hz. Find power ratio between Input & output.

$$f(t) = 5 + 3 \cos 7500\pi t + 6 \sin 10000\pi t + 9 \cos 20000\pi t + 6 \sin 25000\pi t$$

Sol.



$$G(f) = F(f) \cdot H(f)$$



Input power (P(t))

$$P_{av\ In} = \sum_{n=-\infty}^{\infty} |C_n|^2$$

$$= 5^2 + 2(3^2 + 6^2 + 9^2 + 6^2) =$$

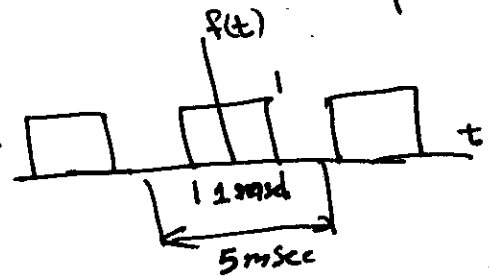
output power

$$P_{av\ out} = 5^2 + 2(3^2 + 6^2) = 115 \text{ watt}$$

$$\frac{P_{output}}{P_{In}} = \frac{115}{115} \times 100\%$$

Exp 7 / Triang of pulse shown $f(t)$ pass through Block $H(t)$ the output $g(t)$. Find ratio between input & output power

$$g(t) = 5 + 4.5 \cos 400\pi t + 3.2 \cos 600\pi t + 2.1 \cos 800\pi t$$



Exp 8 / Draw ^{spectrum} ~~output~~ of LPF (~~cut off frequency~~ $\omega_c = 100 \text{ rad/sec}$)
 If Input frequency Domain $F(\omega) = 1/\sqrt{5 + j\omega^2}$

2. Modulation Techniques

Modulation : Is that process by which a property or a parameter of a signal is varied in proportion to a second signal. The first signal call (Carrier).

Carrier signal $\phi(t) = A_c \cos \omega_c t$ (unmodulation signal)

A_c : Carrier Amplitude (Volt)

f_c : Carrier frequency

$$f_c = \frac{\omega_c}{2\pi} \text{ (Hz)}$$

- Modulation signal

$$\phi(t) = a(t) \cos(\theta(t))$$

- In amplitude modulation (AM), $a(t)$ varies with ~~time~~ input signal when $\theta(t)$ is fixed to carrier frequency $\omega_c t$

$$\phi_{AM}(t) = a(t) \cos(\omega_c t)$$

- In frequency modulation (FM) & phase modulation (PM) call angle modulation, $a(t)$ be constant at carrier Amplitude A_c and $\theta(t)$ will be varies with signal

$$\phi_{\substack{FM \\ PM}}(t) = A_c \cos(\theta(t))$$

2-1 Amplitude Modulation (AM)

Transmitted of low pass signal waveform over the channel without modulation result proper In attundtion Interference & antenna design for that convert Signal Into Band Pass Signal (shift In frequency) by Modulation.

signal: $f(t)$

Carrier: $A_c \cos \omega_c t$

$$\Phi_{AM}(t) = (A_c + f(t)) \cos \omega_c t$$

The carrier Amplitude will by varies with signal.

i - single tone signal

$$\text{If } f(t) = A_m \cos \omega_m t$$

A_m : ~~carrier~~ signal Amplitude

ω_m : signal frequency

Then ~~mod~~ AM signal

$$\begin{aligned} \Phi_{AM}(t) &= (A_c + A_m \cos \omega_m t) \cos \omega_c t \\ &= A_c (1 + m \cos \omega_m t) \cos \omega_c t \end{aligned}$$

ii) The ~~Band~~ Low pass signal

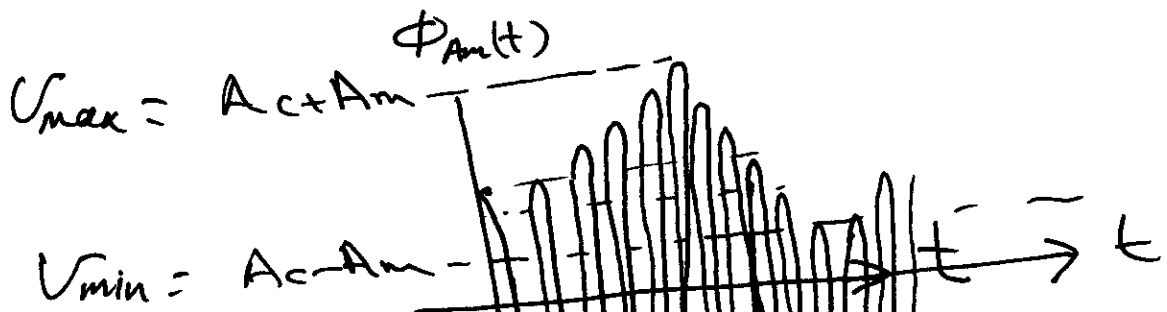
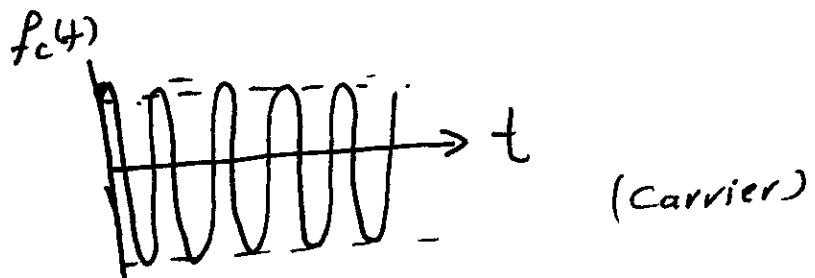
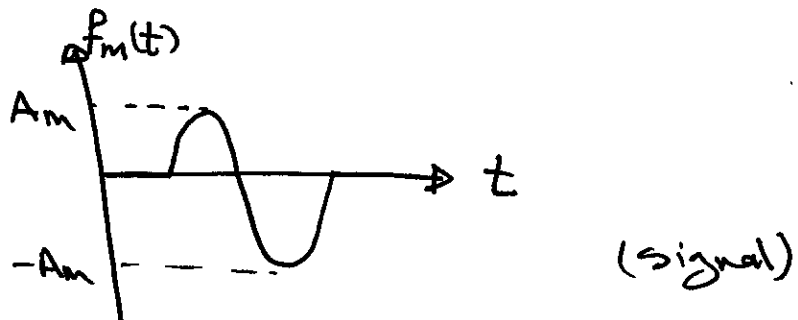
AM modulated signal ($f(t)$)

$$\phi_{AM}(t) = A_c (1 + m f(t)) \cos \omega_c t$$

2-1.1 Time domain AM signal

Let ~~single tone~~ $f(t)$ unmodulated signal

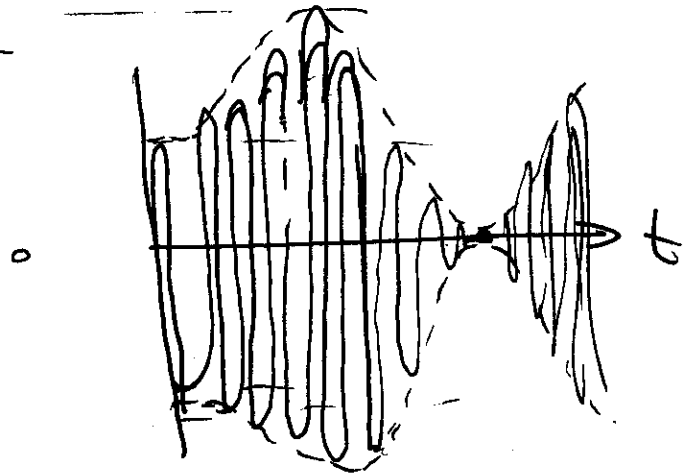
$$f_c(t) = A_c \cos \omega_c t$$



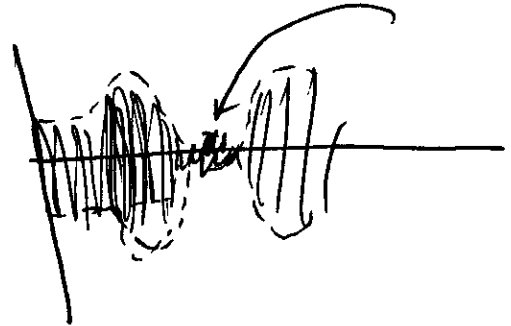
$$A_m = \frac{V_{max} - V_{min}}{2}$$

(AM signal)

\Rightarrow $A_m = A_c$ $m = 1$
 $2A_c$



\Rightarrow $A_m > A_c$ $m > 1$ over/ub



For that $m \leq 1$

A 2-1-2 AM spectrum & Bandwidth

i) For single tone signal $f_m(t) = A_m \cos \omega_m t$

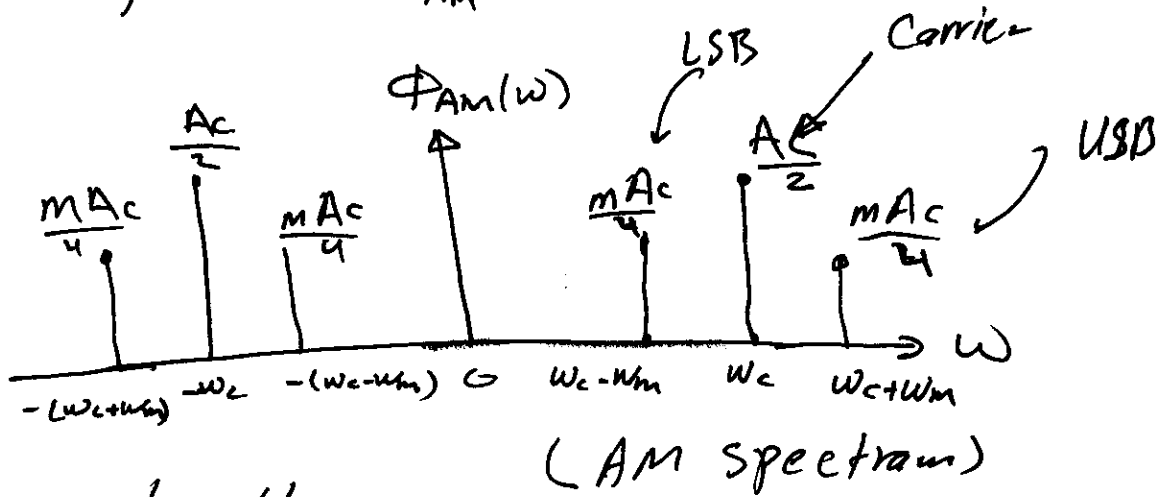
$$\phi_{AM}(t) = A_c (1 + m \cos \omega_m t) \cos \omega_c t$$

$$= A_c \cos \omega_c t + m A_c \cos \omega_c t \cos \omega_m t$$

$$= A_c \cos \omega_c t + \frac{m A_c}{2} \cos(\omega_c + \omega_m)t + \frac{m A_c}{2} \cos(\omega_c - \omega_m)t$$

$\{ BW = 2 f_m \}$ Hz

For that spectrum $\Phi_{AM}(\omega)$ will be



That component call

Carrier $(A_c \cos \omega_c t)$

Upper Side Band $(\frac{m A_c}{2} \cos(\omega_c + \omega_m)t)$ (USB)

Lower Side Band $(\frac{m A_c}{2} \cos(\omega_c - \omega_m)t)$ LSB

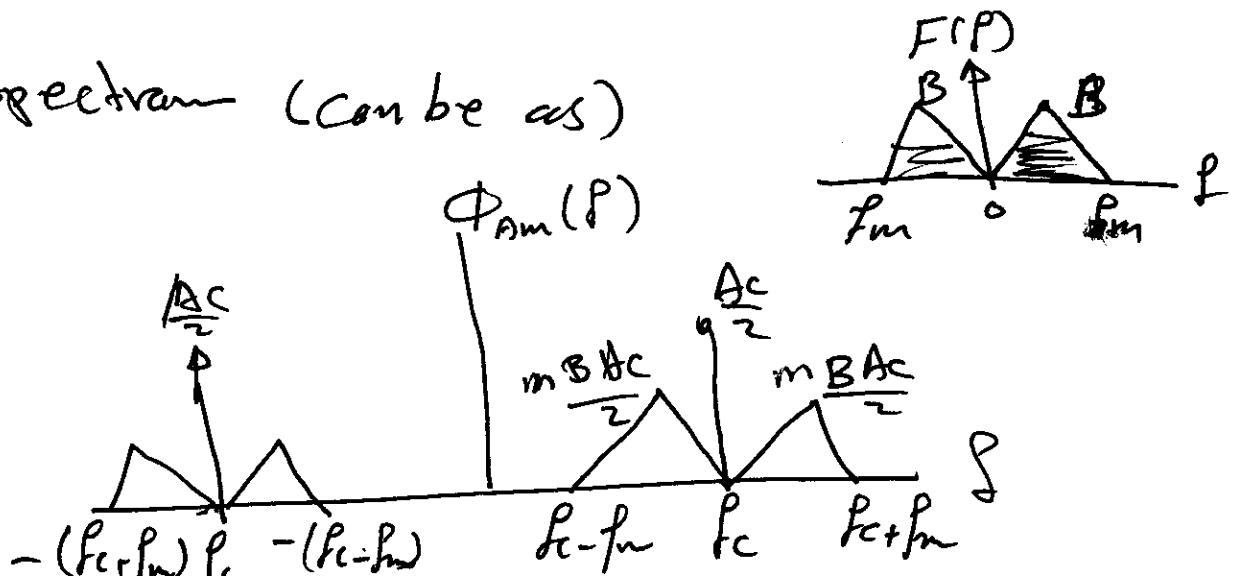
Spectrum for pass band signal (Low pass signal)

signal equation $f(t)$

$$\Phi_{AM}(t) = A_c (1 + m f(t)) \cos \omega_c t$$

$$= A_c \cos \omega_c t + m A_c f(t) \cos \omega_c t$$

signal spectrum (can be as)



2.1.3 AM Power

The AM equation

$$\phi_{AM}(t) = \underbrace{A_c \cos \omega_c t}_{\text{Carrier}} + \underbrace{\frac{mA_c}{2} \cos(\omega_c + \omega_m)t}_{\text{USB}} + \underbrace{\frac{mA_c}{2} \cos(\omega_c - \omega_m)t}_{\text{LSB}}$$

$$P = \frac{V_{rms}^2}{R} = \frac{V_p^2}{2R}$$

Cos or Sine
Signal $V_{rms} = \frac{V_p}{\sqrt{2}}$

- The Carrier Power

$$P_c = \frac{A_c^2}{2R}$$

- Side Band Power (USB OR LSB)

$$P_{side} = \frac{(\frac{mA_c}{2})^2}{2R} = \frac{m^2 A_c^2}{8R} = P_c \frac{m^2}{4}$$

- Sides Bands Power

$$P_{sides} = 2 P_{side} = \frac{m^2 A_c^2}{4R} = P_c \frac{m^2}{2}$$

- Total power

$$P_T = P_c + P_{sides}$$

$$P_T = P_c \left(1 + \frac{m^2}{2}\right)$$

- Modulation efficiency

$$\eta = \frac{P_{sides}}{P_T} = \frac{m^2}{2 + m^2}$$

Hint: If signal consist of multi-tone modulation index used in equation m_{eff} (effective mod. index)

$$m_{eff} = \sqrt{m_1^2 + m_2^2 + \dots}$$

$$m_{eff} = \sqrt{m_1^2 + m_2^2 + \dots}$$

Exp/ an modulated signal $f(t) = 5 \cos 5000\pi t$ modulated
 In AM signal with carrier $f_c(t) = 20 \cos 50000\pi t$

- Find
- ① AM equation
 - ② Draw AM spectrum & find Bandwidth
 - ③ calculate P_c, P_T, m
 - ④ If signal change $f_2(t) = 3 \cos 5000\pi t + \sin$
 $+ 6 \sin 5500\pi t + 10 \cos 5900\pi t$

Find P_T and

Sol

$$\begin{aligned} \Phi_{AM}(t) &= (20 + 5 \cos 5000\pi t) \cos 5\pi \times 10^4 t \\ &= 20 (1 + 0.25 \cos 5000\pi t) \cos 5\pi \times 10^4 t \end{aligned}$$

$$A_c = 20$$

$$m = 0.25$$

$$\omega_c = 5\pi \times 10^4 \text{ rad/sec}$$

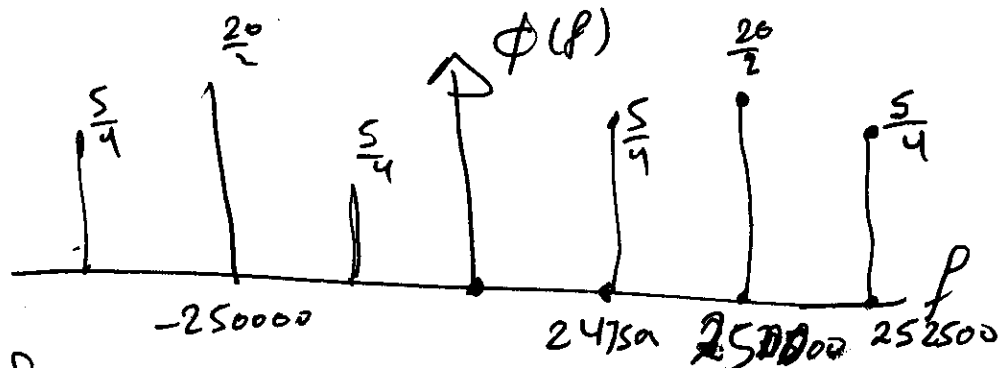
$$\omega_m = 5000\pi \text{ rad/sec}$$

$$f_c = 2.5 \times 10^5 \text{ Hz}$$

$$f_m = 2500 \text{ Hz}$$

② AM spectrum

$$\Phi_{AM}(t) = 20 \cos 5\pi \times 10^4 t + \frac{5}{2} \cos (5 \times 10^4 + 3 \times 10^3)\pi t + \frac{5}{2} \cos (5 \times 10^4 - 3 \times 10^3)\pi t$$



$$BW = 2f_m = 2 \cdot 2500 = 5000 \text{ Hz}$$

$$P_c = \frac{\left(\frac{20}{2}\right)^2}{2R} = 50 \text{ watt}$$

$$R \approx 1$$

$$P_T = P_c (1 + \frac{m^2}{2}) = 51.5625 \text{ watt}$$

$$P_{\text{side}} = 2 P_{\text{USB}} = P_T - P_c = 1.5625 \text{ watt}$$

$$m = \frac{P_{\text{USB}}}{P_T} = \frac{m^2}{2 + m^2} = 1.5625 \%$$

$$(4) f_z(t) = 3 \cos 5000\pi t + 6 \sin 5500\pi t + 10 \cos 5400\pi t$$

$$m_{\text{eff}} = \sqrt{m_1^2 + m_2^2 + m_3^2}$$

$$m_1 = \frac{3}{20} = 0.15$$

$$m_2 = \frac{6}{20} = 0.3$$

$$m_3 = \frac{10}{20} = 0.5$$

$$m_{\text{eff}} = \sqrt{0.15^2 + 0.3^2 + 0.5^2} = 0.602$$

$$P_T = P_c \left(1 + \frac{m_{\text{eff}}^2}{2}\right) = 59.0601 \text{ watt}$$

Exp₁ / an modulating signal shown Find P_T & Draw spec.

$$\phi(t) = 10(1 + 0.2 \sin 5000\pi t) \cos 5000\pi t$$

what type of mod.? why? *

Exp₂ / an AM waveform equal to 500W is 75% modulated Calculate the transmitting power? If antenna Resistance 50 Ω Find all spectrum Component voltage (peak voltage).

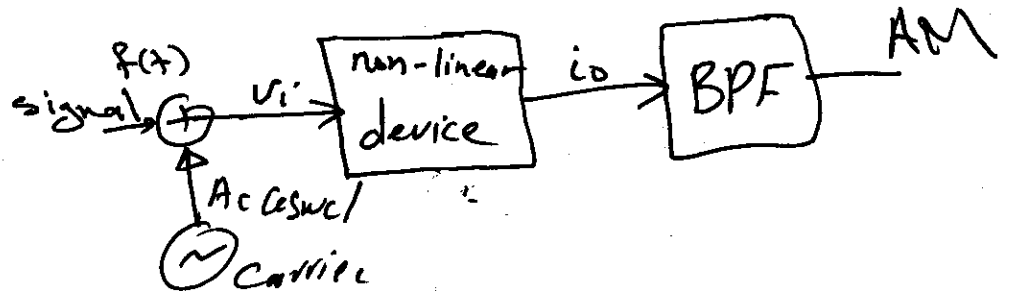
2.14 AM modulator

There are two type of AM generator

- 1- non Linear AM generator (modulator)
- 2- Linear AM generator (chopper)

dependent of element characteristics of material used in circuit.

i) non Linear AM modulator



non linear device equation

$$i_o = a_0 + a_1 V_i + a_2 V_i^2 + \dots$$

a_0, a_1, \dots, a_n : element parameter

Ex/

$$i_o = a_0 + a_1 V_i + a_2 V_i^2$$

$$V_i = f(t) + A_c \cos \omega t$$

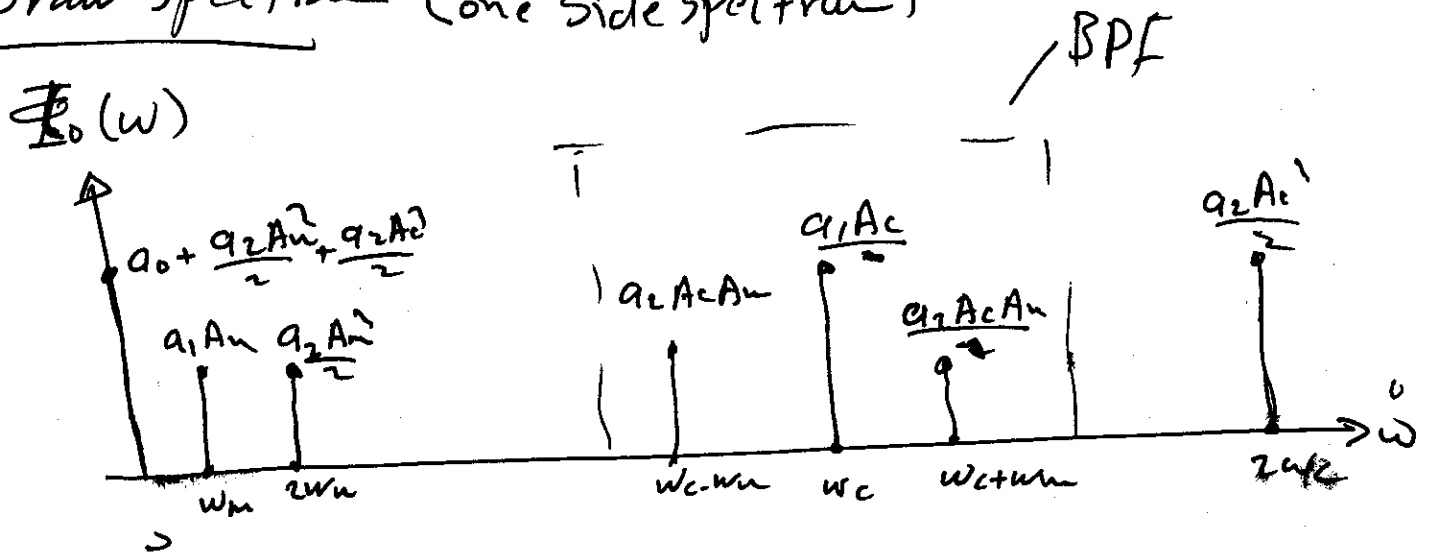
$$i_o = a_0 + a_1 (f(t) + A_c \cos \omega t) + a_2 (f(t) + A_c \cos \omega t)^2$$

let $f(t) = A_m \cos \omega_m t$ single tone

$$i_o = a_0 + a_1 A_m \cos \omega_m t + a_1 A_c \cos \omega t + a_2 (A_m \cos \omega_m t + A_c \cos \omega t)^2$$

$$\begin{aligned}
 i_0 = & a_0 + a_1 A_m \cos \omega_m t + a_1 A_c \cos \omega_c t + \frac{a_2 A_m^2}{2} + \frac{a_2 A_m^2}{2} \cos 2\omega_m t \\
 & + \frac{2a_2 A_c A_m}{2} \cos(\omega_c + \omega_m)t + \frac{2a_2 A_c A_m}{2} \cos(\omega_c - \omega_m)t + \frac{a_2 A_c^2}{2} + \frac{a_2 A_c^2}{2} \\
 & \cos 2\omega_c t
 \end{aligned}$$

(ii) Draw spectrum (one side spectrum)



after used BPF of $f_{\text{center}} = f_c$

$$2f_m \leq \text{BW} < (2f_c - 2f_m)$$

The output will be

$$\begin{aligned}
 v_0(t) = & a_1 A_c + a_2 A_c A_m \cos(\omega_c + \omega_m)t + a_2 A_c A_m \cos(\omega_c - \omega_m)t \\
 = & a_1 A_c \left(1 + \frac{2a_2 A_m}{2a_1} \cos \omega_m t \right) \cos \omega_c t
 \end{aligned}$$

For that modulation index in this case can be known as

2.1.5 - Double side Band SUPPRESSED CARRIER DSB-SC

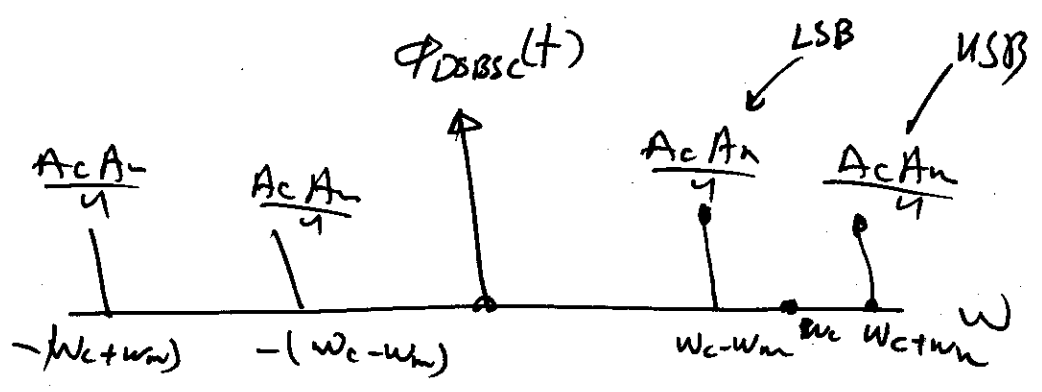
This type of AM contain USB, LSB without carrier because all information held in sides and carrier don't have any information

$$\Phi_{DSBSC}(t) = f(t) * A_c \cos \omega_c t$$

i) Single tone signal

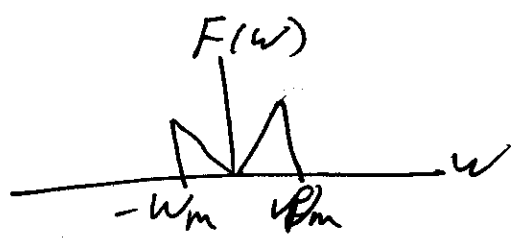
$$f(t) = A_m \cos \omega_m t$$

$$\Phi_{DSBSC}(t) = \frac{A_m A_c}{2} \cos(\omega_c + \omega_m)t + \frac{A_m A_c}{2} \cos(\omega_c - \omega_m)t$$

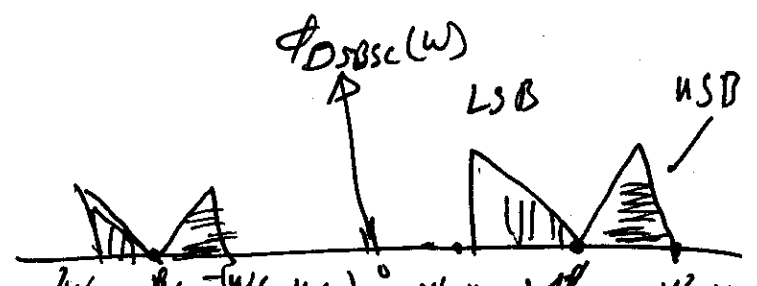


ii) Bandpass signal

$$f(t)$$



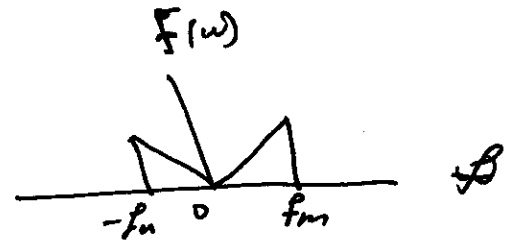
$$\Phi_{DSBSC}(t) = f(t) \cos \omega_c t$$



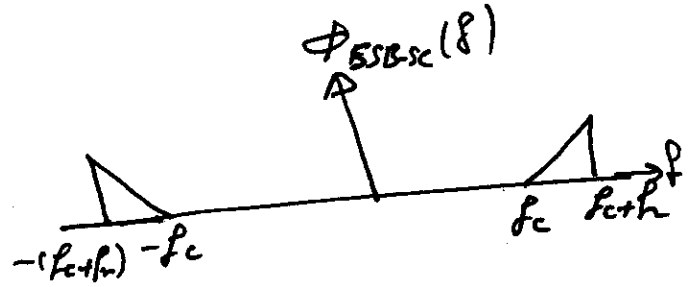
21.6 Single Side Band Suppressed Carrier (SSB-SC)

This type of AM contains only at USB or LSB only by this way power will be used to transmit one side with min Bandwidth To transmitted signal.

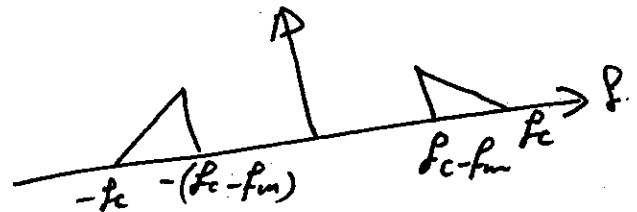
SSB-SC spectrum



SSB-SC (USB)



SSB-SC (LSB)



$$\Phi_{SSB-SC}(t) = f(t) \cos \omega_c t + \dot{f}(t) \sin \omega_c t \quad (\text{USB})$$

$$\Phi_{SSB-SC}(t) = f(t) \cos \omega_c t - \dot{f}(t) \sin \omega_c t \quad (\text{LSB})$$

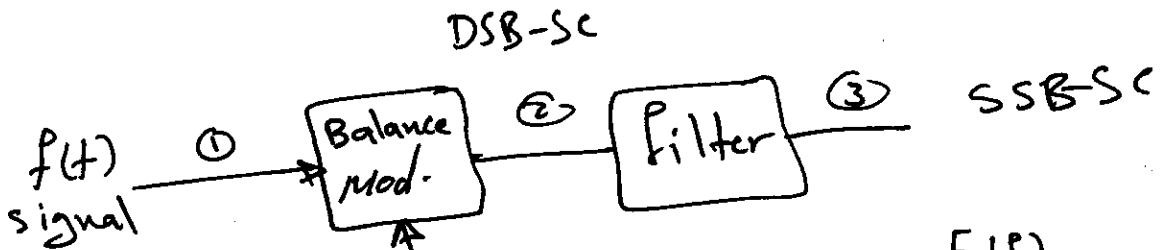
$$BW_{SSB-SC} = f_m$$

2.1.6.1 SSB-SC Modulation

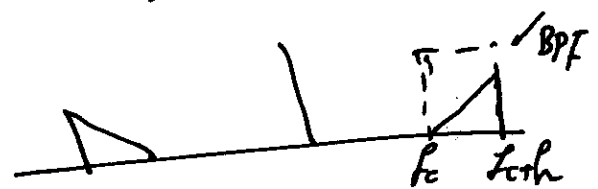
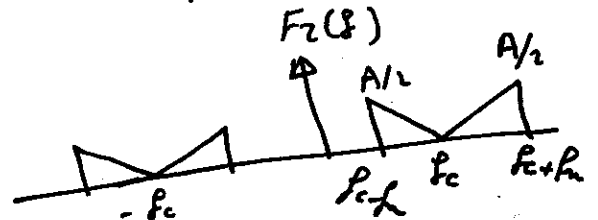
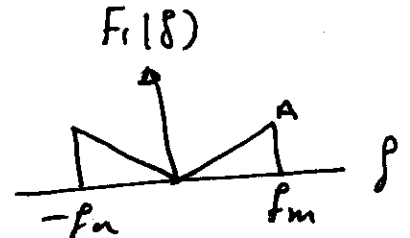
There are two methods

i- Filter method

used DSB-SC modulator than used filter to select one ~~side~~ side



$$f_2(t) = f(t) \cos \omega_c t$$



Generate SSB-SC

$$f_3(t) =$$

USB

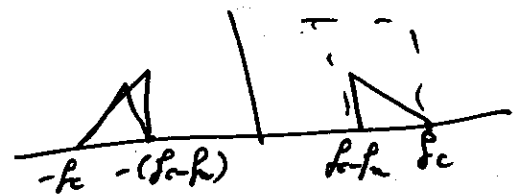
BPF of $f_{center} = f_1 = f_c$
 $f_2 = f_c + f_m$

OR used HPF $f_c \geq f_c$

(LSB)

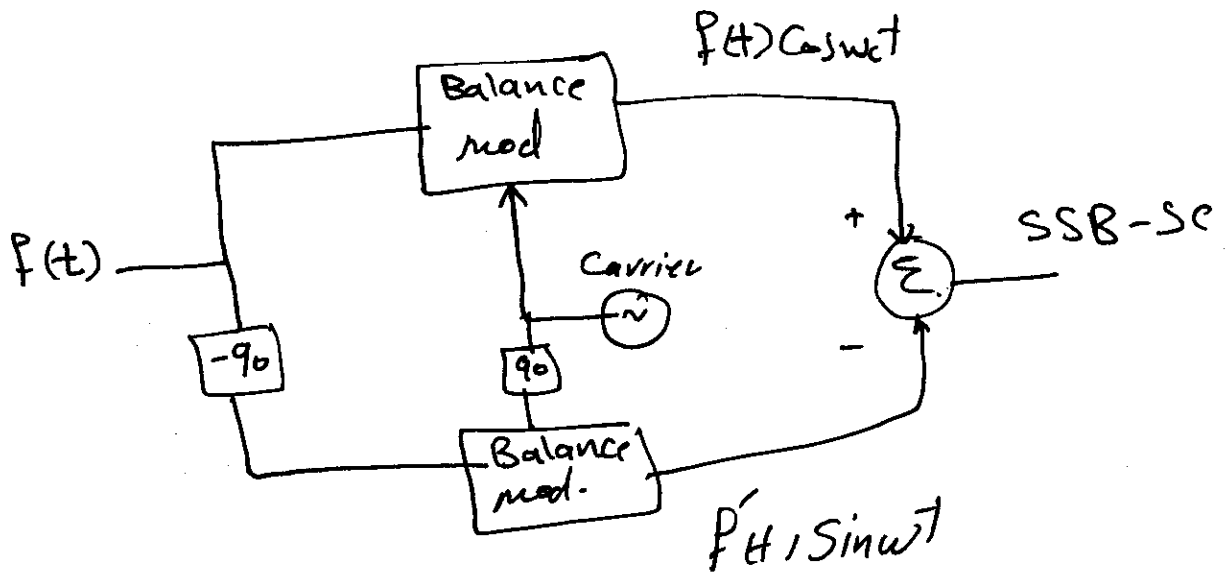
BPF of $f_1 = f_c - f_m$
 $f_2 = f_c$

OR LBF $f_{cut} = f_c$



BW = f_m

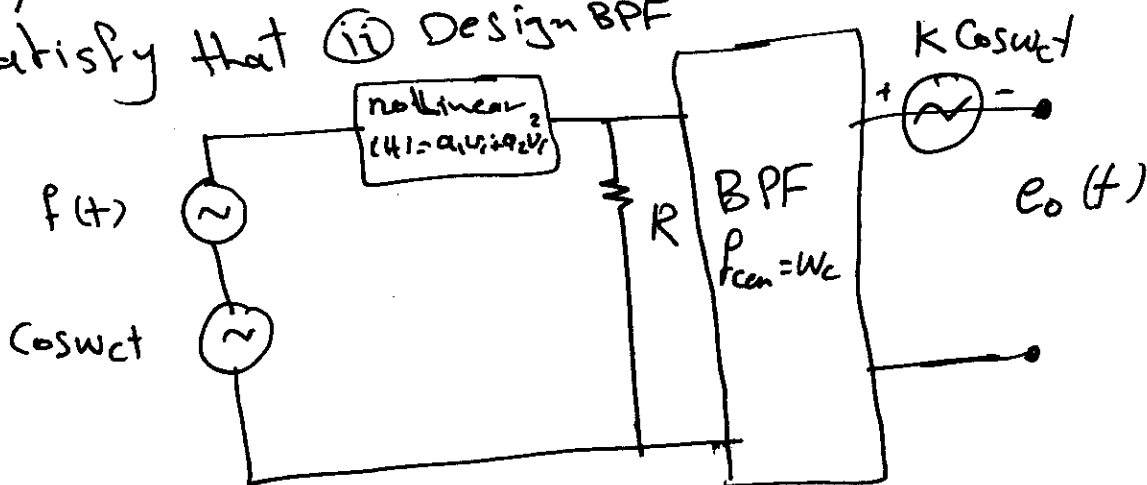
ii) phase shift methods



$$\Phi_{USB} = F(t) \cos \omega_c t + F'(t) \sin \omega_c t$$

$$\Phi_{LSB} = F(t) \cos \omega_c t - F'(t) \sin \omega_c t$$

(i) Exp/ The model to generate DSB-SC signal value of k to satisfy that (ii) Design BPF

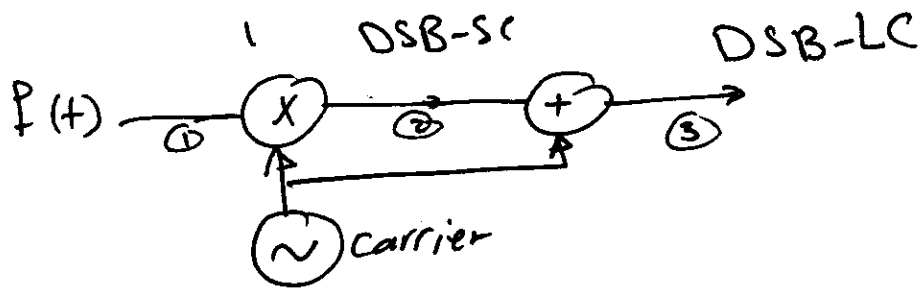


Exp₂/ SSB-SC Modulator (Filter method) Design all circuit & filter to generate USB IF Input signal

$$f(t) = 2 \cos 500\pi t + 3 \cos 1000\pi t$$

2.17 Generation of DSB-LC (Double Side Band - Large Carrier)

First generate DSB-SC then add some carrier
 This carrier help Receiver to synchronise the carrier
 Receiver

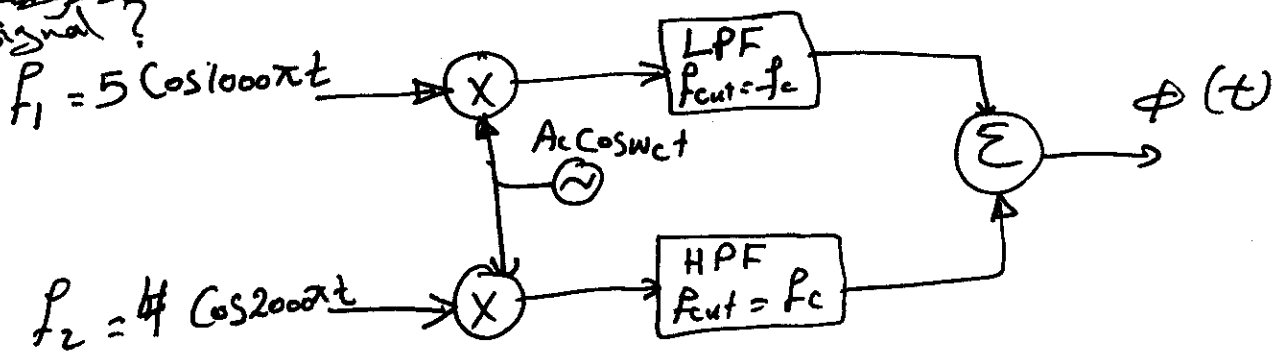


① $f_1(t)$

$f_2(t) = A_c f(t) \cos \omega_c t$

$f_3(t) = A_c f(t) \cos \omega_c t + A_c \cos \omega_c t = A_c (1 + f(t)) \cos \omega_c t$

Exp/ The system shown below generated two signal of SSB-SC can transmitted at the same channel find spectrum ~~every~~ at every point ①, ②, ③, ④, ⑤ & ⑥ + time domain signal?



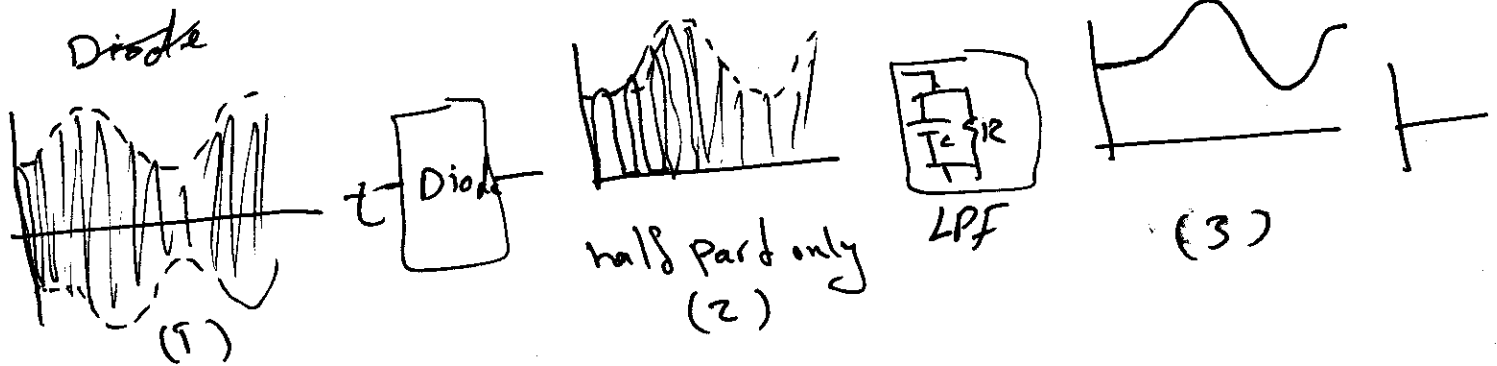
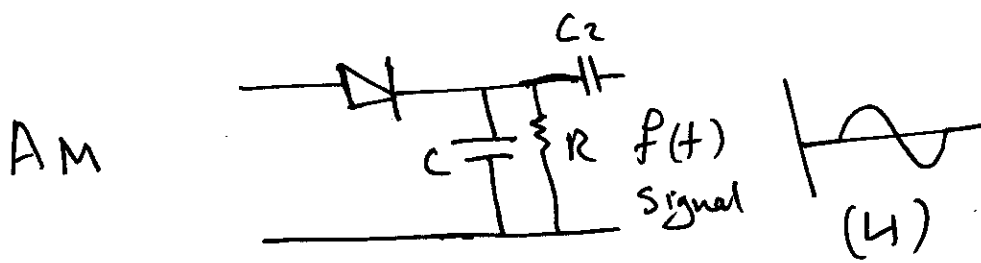
2-18AM - Demodulator

This circuit used In Receiver and can detect signal (message) from modulated signal.

There are two method to detect AM signal and other type of AM (DSB-SC, SSB-SC, DSB-LC)

i-non-Linear demodulator or (Square Law detector)

This method used diode (non linear device) & LPF as shown

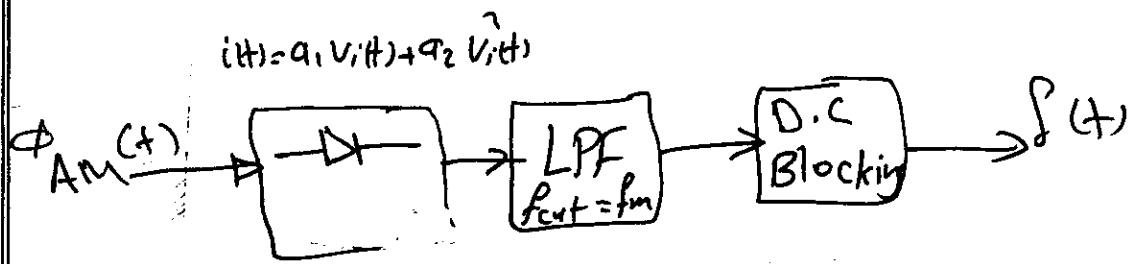


- by diode half part of signal used
 - by RC (LPF) charge and discharge signal to generate low pass signal

- by used C_2 (D.C Blocking) Removed

D.C signal from result signal

Nonlinear demodulator Block Diagram

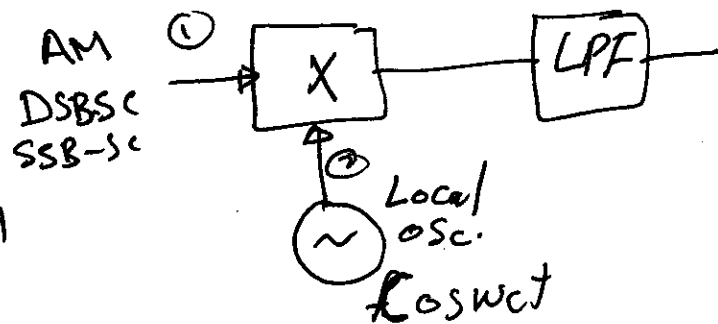


Exp/ For the Block Diagram shown upper I
 Input signal $\phi_{AM}(t) = A_c (1 + m \cos \omega_m t) \cos \omega_c t$
 Prove all part work? by spectrum Drawing

Exp₂/ Prove by using nonlinear device can not detect DSB-SC

ii) Synchronous Demodulator

This type of demodulator used Mixer to detect signal and can detect AM, DSB-SC OR SSB-SC

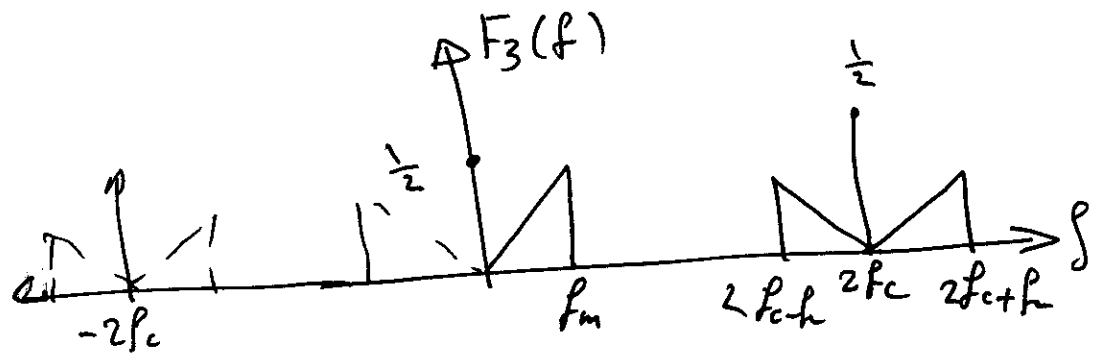


$$P_1(t) = A_c \cos \omega_c t + f(t) \cos \omega_c t$$

$$P_2(t) = \cos \omega_c t$$

$$P_3(t) = P_1(t) \cdot P_2(t)$$

$$(\cos \omega_c t + f(t) \cos \omega_c t) \cdot \cos \omega_c t = \frac{1}{2} + \frac{1}{2} \cos 2\omega_c t + \frac{f(t)}{2} + \frac{f(t)}{2}$$

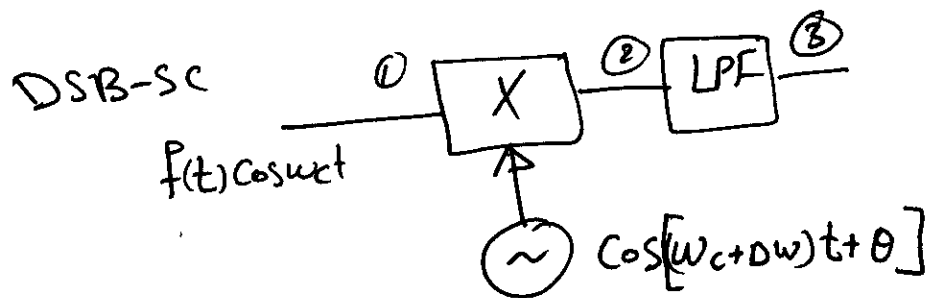


by using LPF $f_m \leq f_{cut} < 2f_c - f_m$

can reject signal

$$f_u(t) = \frac{1}{2} + \frac{1}{2} f(t)$$

This Modulation call Synchronous because the carrier generated from Local osc. must by equal In frequency and phase different to the modulation carrier



$$f_1(t) = f(t) \cos wct$$

$$f_2(t) = f(t) \cos wct \cdot \cos(wc+dw)t+\theta$$

$$= \frac{1}{2} f(t) \cos(\Delta\omega t + \theta) + \frac{1}{2} f(t) \cos[(2w_c + \Delta\omega)t + \theta]$$

after LPF

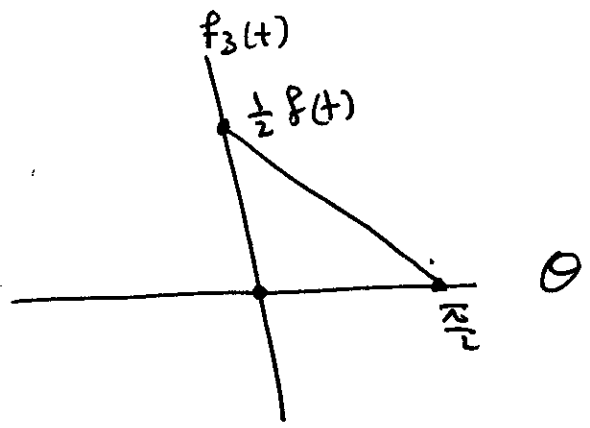
$$f_3(t) = \frac{1}{2} f(t) \cos(\Delta\omega t + \theta)$$

@ If $\Delta\omega = 0$ phase different only

$$f_3(t) = \frac{1}{2} f(t) \cos \theta$$

If phase different by $\theta = \frac{\pi}{2}$ the output = 0

$$f_3(t) = 0$$

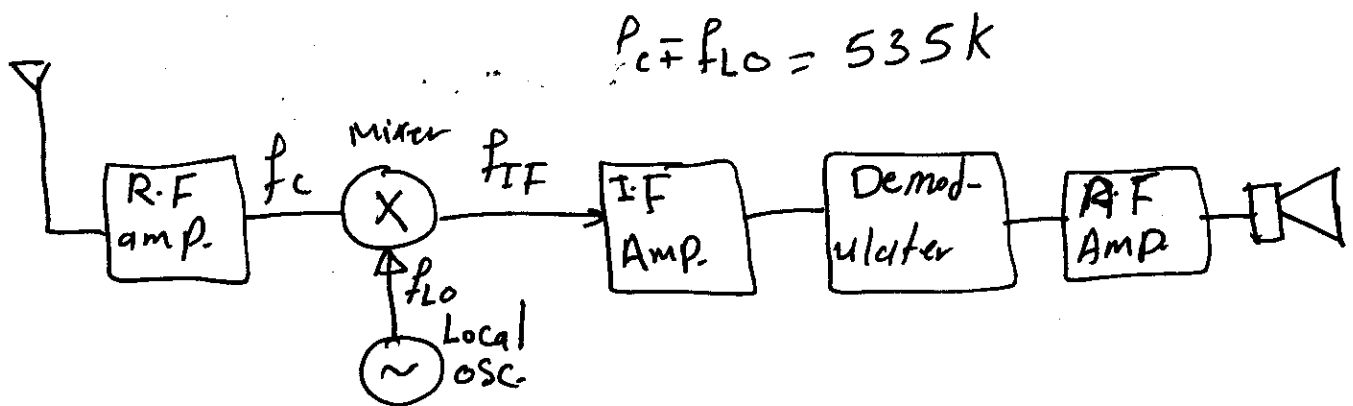


② $\theta = 0$ only different in freq. $\Delta\omega$

$$f_3(t) = \frac{1}{2} P(t) \cos \Delta\omega t$$

this $\cos \Delta\omega t$ will reduce and attenuate the output by noise.

AM Receiver Block Diagram



Three stage

- ① R F (Radio frequency) f_c
- ② IF (Intermidant freq) $f_{IF} = 535 \text{ kHz}$

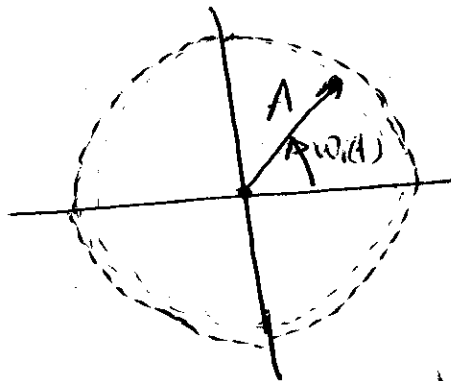
$$f_{IF} = f_c + f_{\text{Local osc.}}$$

③ A.F. (Audio frequency) hair signal

3 - Angle Modulation

In angle modulation the phase angle is varied in proportion to $f(t)$ (signal).

The phasor representation of a constant-amplitude sinusoid is shown below.



If the angular rate is not constant, we can still write a relation between the instantaneous angular rate $\omega(t)$ and $\theta(t)$

$$\omega(t) = \frac{d\theta}{dt} \Rightarrow \theta(t) = \int \omega(t) dt$$

In Angle modulation modulated signal equation

$$\phi(t) = A_c \cos \theta(t)$$

The Instantaneous frequency $\omega(t)$

3.1 FM (Frequency Mod.)

$$\omega(t) = \omega_c + K_f f(t)$$

For FM

K_f : FM constant ~~with units~~ rad/volt-sec

If $f(t) = A_m \cos \omega_m t$ (single tone)

$$\omega(t) = \omega_c + K_f A_m \cos \omega_m t$$

$$\theta(t) = \int w_i(t) dt$$

$$\theta(t) = \omega_c t + \frac{k_f A_m}{\omega_m} \sin \omega_m t$$

Then the FM equation

$$\phi_{FM}(t) = A_c \cos[\omega_c t + m_f \sin \omega_m t]$$

$m_f = \text{modulation Index} = \frac{\Delta f}{f_m} = m_f$

Δf : frequency deviation

$$2\pi \Delta f = k_f \cdot A_m$$

$$\Delta \omega = k_f A_m$$

General FM equation

$$\phi_{FM}(t) = A_c \cos[\omega_c t + k_f \int f(t) dt]$$

3.12 PM (Phase Modulation)

k_p : PM constant $\theta(t) = \omega_c t + k_p \phi(t)$

For single tone

$$\phi_{PM}(t) = A_c \cos[\omega_c t + \cancel{m_f} \cos \omega_m t]$$

$\Delta \phi$: ~~phase deviation~~
max phase deviation

$$\Delta \phi = k_p \cdot A_m$$

General Equation of PM

$$\phi_{PM}(t) = A_c \cos[\omega_c t + k_p \phi(t)]$$

Example 1 an angle modulator (FM, PM) of ~~carrier~~ unmodulated carrier $f_c(t) = 100 \cos 50 \times 10^3 \pi t$ and Input signal $f_m(t) = 3 \cos 5000 \pi t$ If modulator have $K_f = 500 \pi$ rad/volt $K_p = 100$ Find FM & PM equation

Sol.

FM output

$$f_m(t) = 3 \cos 5000 \pi t$$

$$\omega_i(t) = \omega_c + K_f f_m(t) \\ = 50 \times 10^3 \pi + 500 \pi * 3 \cos 5000 \pi t$$

$$\theta_i(t) = 50 \times 10^3 \pi t + \frac{1500 \pi}{5000 \pi} \sin 5000 \pi t$$

$$\phi_{FM}(t) = 100 \cos \left[50 \times 10^3 \pi t + 0.3 \sin 5000 \pi t \right]$$

$$m_f = 0.3$$

PM output

$$\theta_i(t) = \omega_c t + K_p \cdot f_m(t)$$

$$= 50 \times 10^3 \pi t + 100 * 3 \cos 5000 \pi t$$

$$= 50 \times 10^3 \pi t + 300 \cos 5000 \pi t$$

$$\phi_{PM}(t) = 100 \cos \left[50 \times 10^3 \pi t + 300 \cos 5000 \pi t \right]$$

$$m_p = 30$$

Q1 / Find the outputs of that systems If signal by ① $f_m(t) = 10 e^{-50t}$ ② $f_m(t) = 30t^2 + 5t + 3$ then Find Instantaneous frequency & phase at time $t = 5 \text{ msec}$.

Example 3

An FM modulator have Input signal voltage 5 volt and 3 kHz have max. frequency deviation 30 kHz. If the signal change as shown calculate modulation index

- ① signal frequency doubled
- ② signal voltage by 15 volt
- ③ signal voltage 15 volt signal freq. 10 kHz

Sol
Hint

from the equation $\Delta f = \frac{k_f A_m}{2\pi}$ Hz
 Δf independent of signal frequency f_m
but on signal amplitude only

$$\textcircled{1} m_f = \frac{\Delta f}{f_m} = \frac{30k}{6k} = 5$$

$$\textcircled{2} k_f = \frac{\Delta f \times 2\pi}{A_m} = \frac{30k \times 2\pi}{5 \text{ volt}} = 12k\pi$$

$$\Delta f_{\text{new}} = \frac{k_f \cdot A_m}{2\pi} = \frac{12k\pi \times 15}{2\pi} = 90 \text{ kHz}$$

$$m_f = \frac{\Delta f}{f_m} = \frac{90k}{3k} = 30$$

$$\textcircled{3} \Delta f_{\text{new}} \text{ for } A_m = 15V = 90 \text{ kHz}$$

$$m_f = \frac{90k}{10k} = 9$$

3.2 Narrowband FM

For ~~small~~ small value of modulation index m_f

If single tone $P(t) = A_m \cos \omega_m t$

$$\phi_{FM}(t) = A_c \cos[\omega_c t + m_f \sin \omega_m t]$$

$$\phi_{FM}(t) = A_c [\cos \omega_c t \cos[m_f \sin \omega_m t] + \sin \omega_c t \sin[m_f \sin \omega_m t]]$$

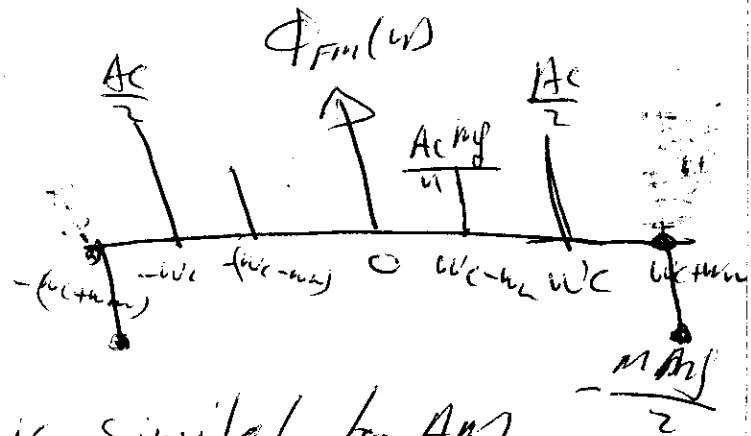
For small m_f

$$\begin{aligned} \cos \theta &= 1 \\ \sin \theta &= \theta \end{aligned} \quad \left. \vphantom{\begin{aligned} \cos \theta &= 1 \\ \sin \theta &= \theta \end{aligned}} \right\} \lim_{\theta \rightarrow 0}$$

$$\phi_{FM}(t) = A_c \cos \omega_c t + A_c \sin \omega_c t \cdot m_f \sin \omega_m t$$

$$= A_c \cos \omega_c t + \frac{A_c m_f}{2} \cos(\omega_c t - \omega_m t) - \frac{A_c m_f}{2} \cos(\omega_c t + \omega_m t)$$

Spectrum of NBFM

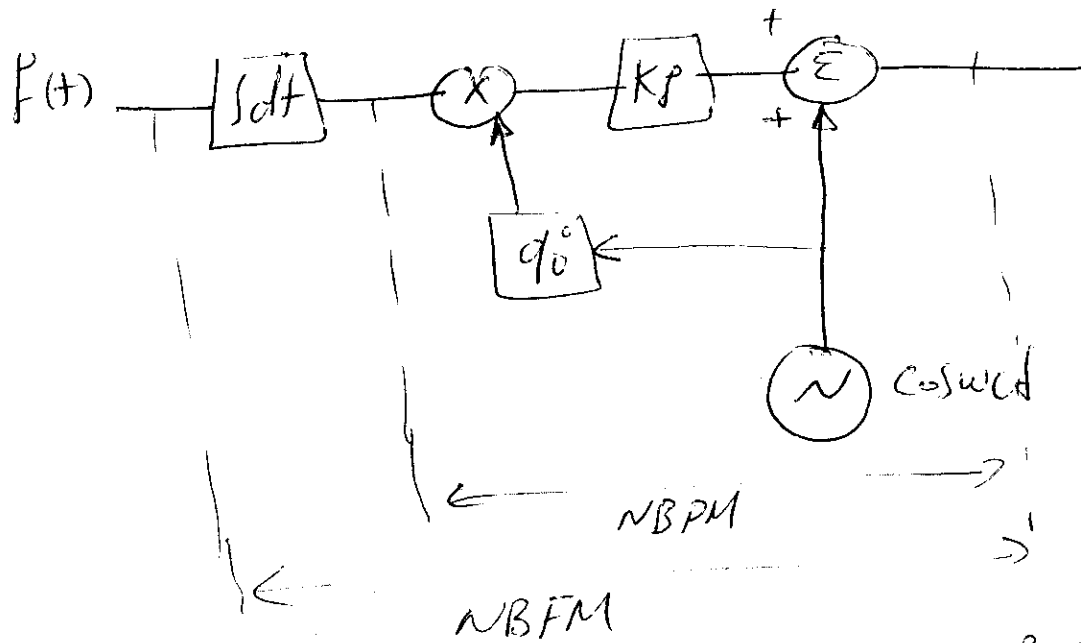


For flat spectrum of NBFM is similar to AM
 expect phase shift in USB by 90°

$$BW_{NBFM} = 2 f_m \text{ (Hz)}$$

$$\text{for } m_f < 0.2$$

The FM (NBFM) generator



This system work to generate NBFM If Integration
 Remove the circuit by NBFM If remove 90° phase
 shift by DSB-SC (AM).

3.3 wide Band Frequency Modulation WBFM

The FM signal

$$\phi_{FM}(t) = A_c [\cos \omega_c t + m_f \sin \omega_m t]$$

Expanding the exponential modulation term in series

$$\phi_{FM}(t) = A e^{j \omega_c t} \left(1 + j m_f \sin \omega_m t - \frac{1}{2!} m_f^2 \sin^2(\omega_m t) - j \frac{1}{3!} m_f^3 \sin^3 \omega_m t + \dots \right)$$

at using numerical solution

$$\phi_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(m_f) \cos(\omega_c t + n\omega_m t)$$

where $J_n(m_f)$ is Bessel function of order depends on m_f (modulation index)

$$\phi_{FM}(t) = A_c J_0(m_f) \cos \omega_c t + J_1(m_f) [\cos(\omega_c + \omega_m t) - \cos(\omega_c - \omega_m t)] \\ + J_2(m_f) [\cos(\omega_c + 2\omega_m t) - \cos(\omega_c - 2\omega_m t)] \\ \vdots$$

all value of $|J_n(m_f)| \geq 0.01$ when the other neglected to approximate signal by 1%

~~equation for Bessel function~~

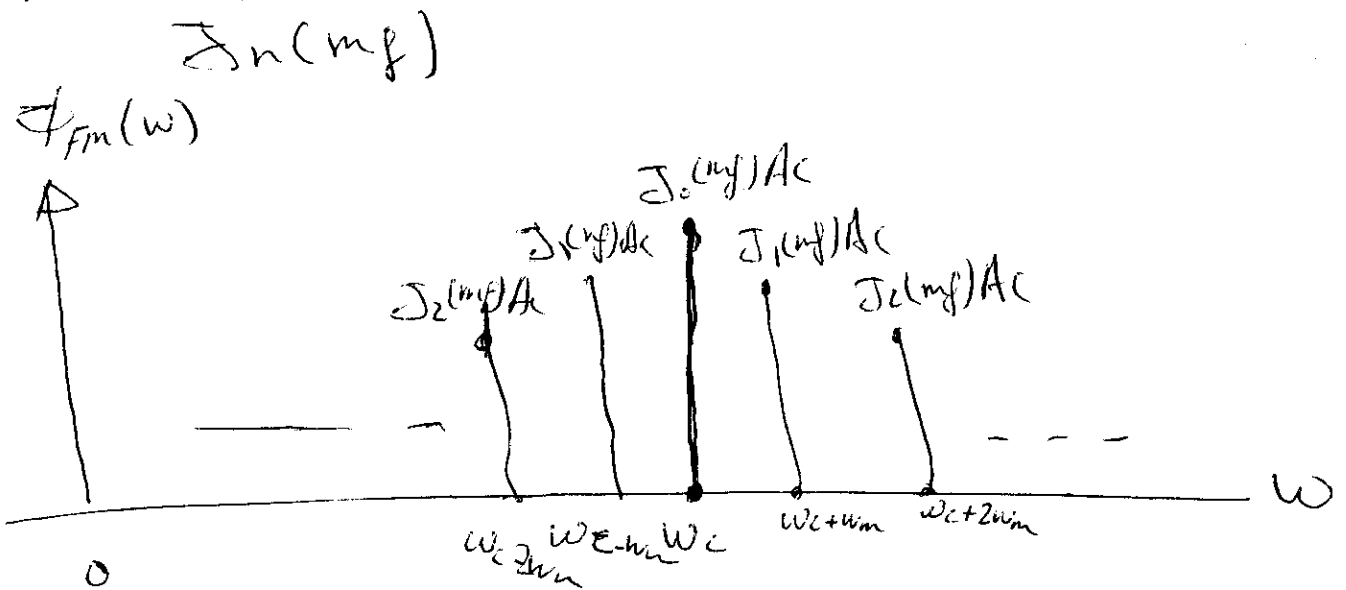
The characteristics of Bessel function?

- 1- $J_n(m_f)$ are real valued
- 2- $J_n(m_f) = J_{-n}(m_f)$ for even
- 3- $J_n(m_f) = -J_{-n}(m_f)$ for odd
- 4- $\sum_{n=-\infty}^{\infty} J_n^2(m_f) = 1$

as shown in figure (6-9) P-292 stremial

3.4 FM Spectrum

The FM spectrum depends on m_f and Bessel value



Exact FM Bandwidth

$$BW_{\text{exact}} = 2 \cdot N \cdot f_m$$

N : max value of m_f

approximate FM BW

$m_f < 0.2$ for NBFM

$$BW = 2f_m$$

$0.2 < m_f < 30$ for WBFM

$$BW \approx 2f_m (M_f + 1)$$

for large m_f

$$BW = 2 \Delta f$$

Average Power in FM

The single tone signal have modulating signal

$$\phi_{FM}(t) = A_c \cos(\omega_c t + m_f \sin \omega_m t)$$

by using series analysis

$$\phi_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(m_f) \cos(\omega_c + n\omega_m)t$$

- The unmodulated carrier power

$$P_{Cun.} = \frac{A_c^2}{2R}$$

- when modulated signal power

$$P_T = P_{Cun.} \left[J_0^2(m_f) + 2(J_1^2(m_f) + J_2^2(m_f) + \dots) \right]$$

when $\sum_{n=-\infty}^{\infty} J_n^2(m_f) \approx 1$

$$P_T \approx P_{Cun.}$$

EXP. / show that r.m.s value of FM signal by

$$\phi_{rms}(t) = \sqrt{\overline{\phi^2(t)}} = A_c \sqrt{\frac{J_0^2(m_f) + 2 \sum_{n=1}^{\infty} J_n^2(m_f)}{2}}$$

Exp/ an FM signal shown

$$\Phi_{FM}(t) = 10 \cos[10^6 \pi t + 3.8 \cos 4000 \pi t]$$

- ① Draw modulation signal spectrum
- ② Calculate modulation index and ~~max~~ max. freq. deviation
- ③ Exact Band width and approximat Band width
- ④ unmodulated carrier power into 50 Ω resistive load
- ⑤ second and third ~~harmonic~~ ^{Sides} Power
- ⑥ Peak voltage of the ~~fourth~~ ^{fourth} side harmonic

Sol.

$$m_f = 3.8$$

$$J_0(3.8) = -0.4$$

$$J_1(3.8) = 0.01$$

$$J_2(3.8) = 0.41$$

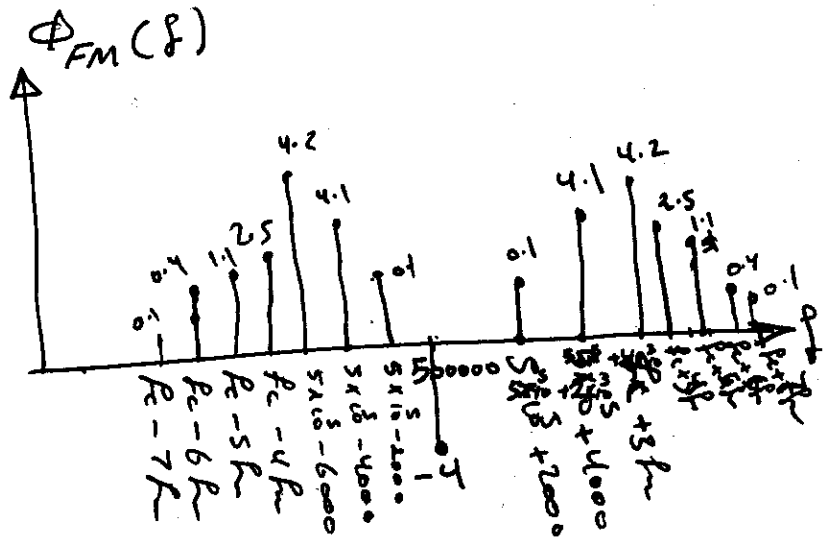
$$J_3(3.8) = 0.42$$

$$J_4(3.8) = 0.25$$

$$J_5(3.8) = 0.11$$

$$J_6(3.8) = 0.04$$

$$J_7(3.8) = 0.01$$



②

$$m_f = 3.8$$

$$\Delta f = m_f \cdot f_m = 3.8 \times 2000 = 7.6 \text{ kHz}$$

$$\text{③ } BW_{\text{exact}} = 2 f_m \cdot n = 2 \times 2000 \times 7 = 28000 \text{ Hz} = 28 \text{ kHz}$$

$$BW_{\text{approximated}} = 2 f_m (\beta + 1) = 2 \times 2 \text{ K} (3.8 + 1) = 19.2 \text{ kHz}$$

$$\text{④ } P_{C_{un.}} = \frac{A_c^2}{2R} = \frac{(10)^2}{2 \times 50} = 1 \text{ watt} \quad (53)$$

$$\text{⑤ } P_{1+P_2} = 2 \cdot P \left[J_2^2(3.8) + J_3^2(3.8) \right]$$

6) $V_{\text{Fourth}} = A_c J_4(3.8)$ (peak)
 $= 10 * 0.25$

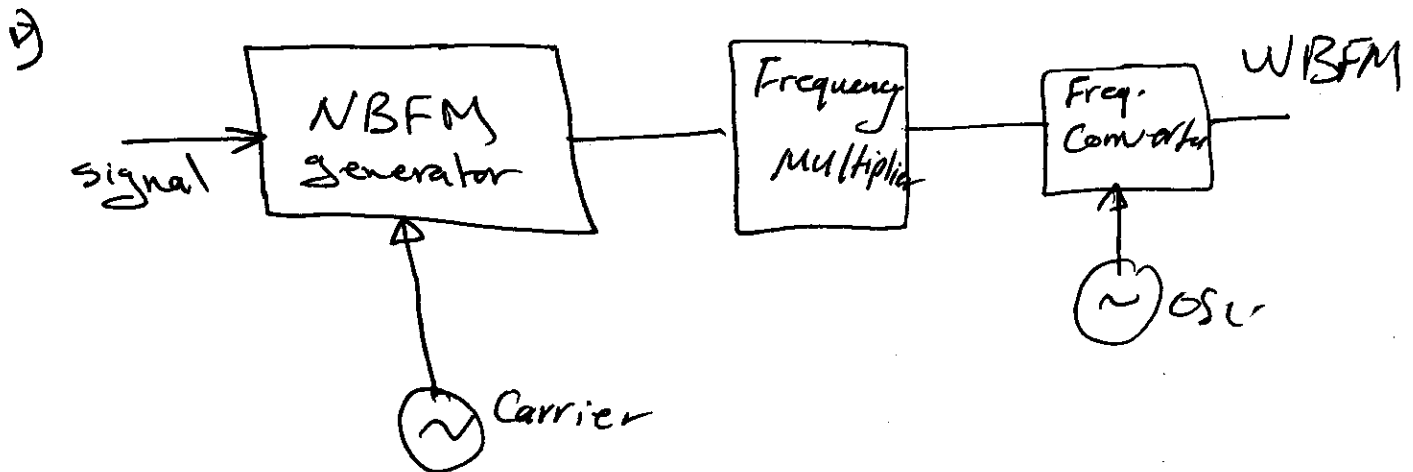
$V_{\text{Fourth}} = 2.5 \text{ volt}$

Generation of WBFM

There are two methods to generate WBFM (wide Band Frequency Modulation)

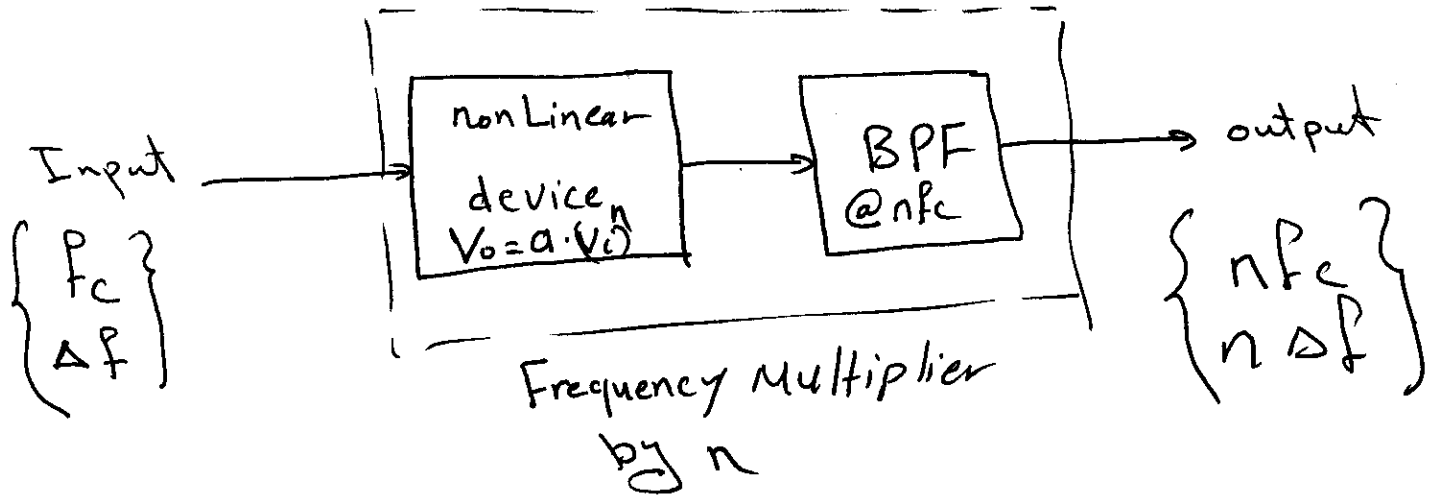
i) Indirect method

By this method NBFM is generated then increasing bandwidth by using (Frequency Multiplier) and shifted carrier by (frequency Converter) to generate WBFM.



1) Frequency Multiplier

It's non linear device and BPF that used to multiply frequency (Carrier) by a factor (n)



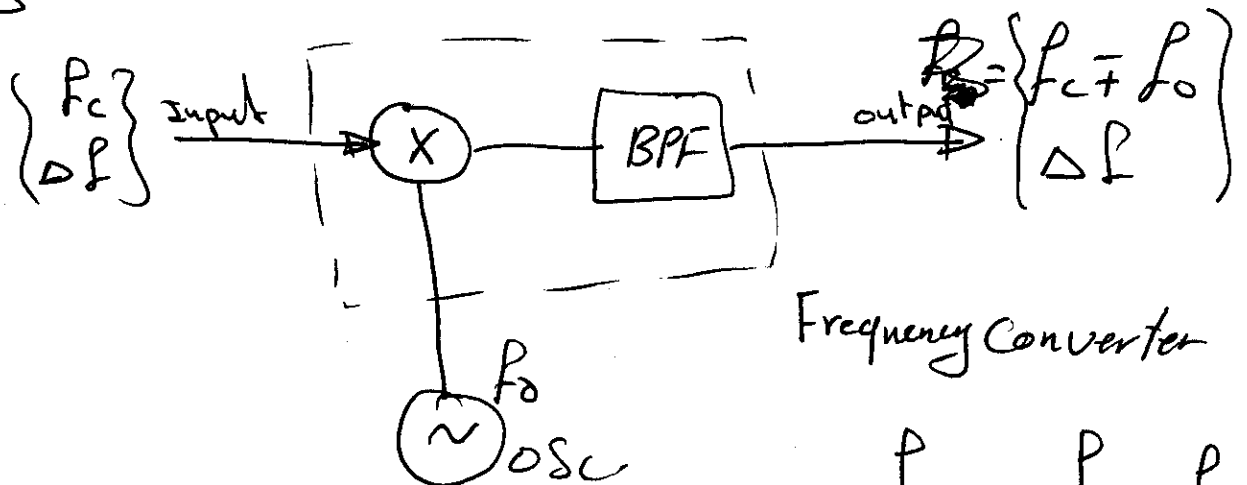
both (f_c & Δf) by growing and multiply by (n)

$$f_{c\text{out}} = n f_{c\text{in}}$$

$$\Delta f_{\text{out}} = n \Delta f_{\text{in}}$$

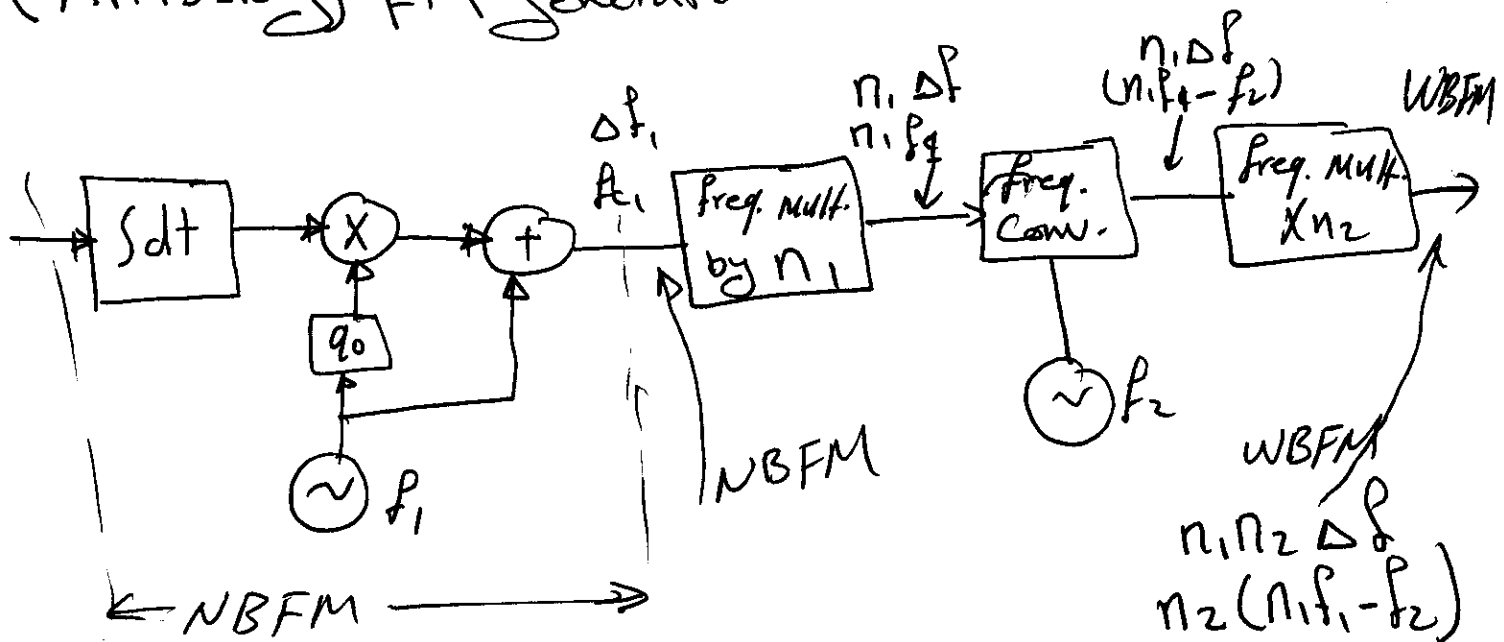
2) Frequency Converter

This device convert carrier of signal to new freq. by using oscillator.



$$f_{c\text{out}} = f_{c\text{in}} + f_0$$

The Block diagram of an Indirect FM generator (Armstrong) FM generator shown below



ii) Direct FM Modulator

In the direct method of generating FM the modulating signal directly controls the carrier frequency. An attempt is usually to generate WBFM.

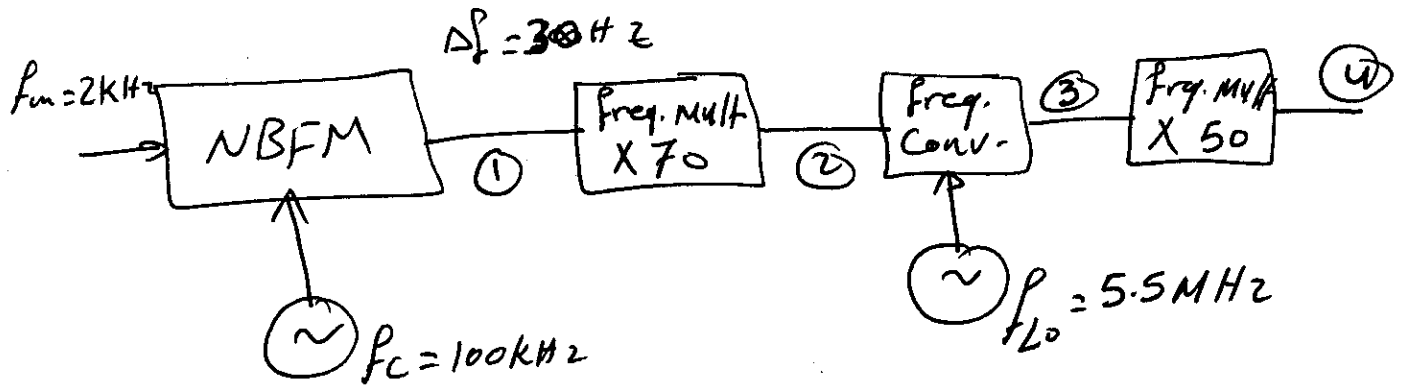
To generate FM direct used Inductive and capacitor (oscillator)

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Then control by input signal at capacitor to generate FM signal. Examples used

- 1) Varactor diode
- 2) FET

Example For the Block Diagram shown Find carrier frequency, Δf , and Bandwidth If signal frequency $f_m = 2\text{ kHz}$



Sol.

① Point (1)

$$\Delta f = 30 \text{ Hz}$$

$$f_c = 100 \text{ kHz}$$

$$BW = 2 f_m = 4 \text{ kHz}$$

$$m_f = \frac{30}{2\text{K}} = 0.015$$

(NBFM)

② $\Delta f = 70 \times 30 = 2.1 \text{ kHz}$

$$f_c = 70 \times 100\text{K} = 7 \text{ MHz}$$

$$m_f = 1.05$$

$$BW = 2 f_m (m_f + 1) = 8.2 \text{ kHz}$$

③ $\Delta f = 2.1 \text{ kHz}$

$$f_{c3} = f_{c2} - f_{L0} = 1.5 \text{ MHz}$$

$$m_f = 1.05$$

$$BW = 8.2 \text{ kHz}$$

④

$$\Delta f = 50 \times 2.1\text{K} = 105 \text{ kHz}$$

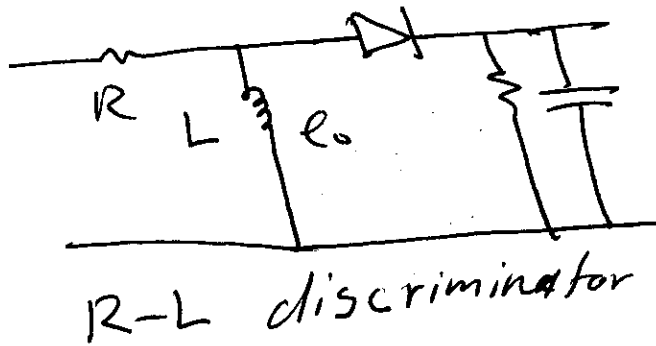
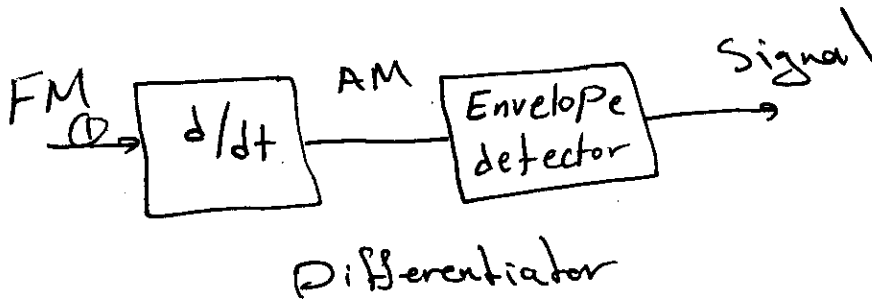
$$f_{c4} = f_{c3} \times 50 = 75 \text{ MHz}$$

Demodulation of FM signal

There are two methods, depend on type of detection

i) Direct method

one method is to use some system which has a linear frequency-to-voltage transfer characteristic call frequency Discriminator.



Point 1 FM equation

$$\phi_{FM}(t) = A_c \cos[\omega_c t + K_f \int f(t) dt]$$

Point 2

$$\frac{d\phi_{FM}(t)}{dt} = -A_c [\omega_c + K_f f(t)] \sin[\omega_c t + K_f \int f(t) dt]$$

This signal $(\frac{d\phi_{FM}(t)}{dt})$ contain information variance

In amplitude & frequency

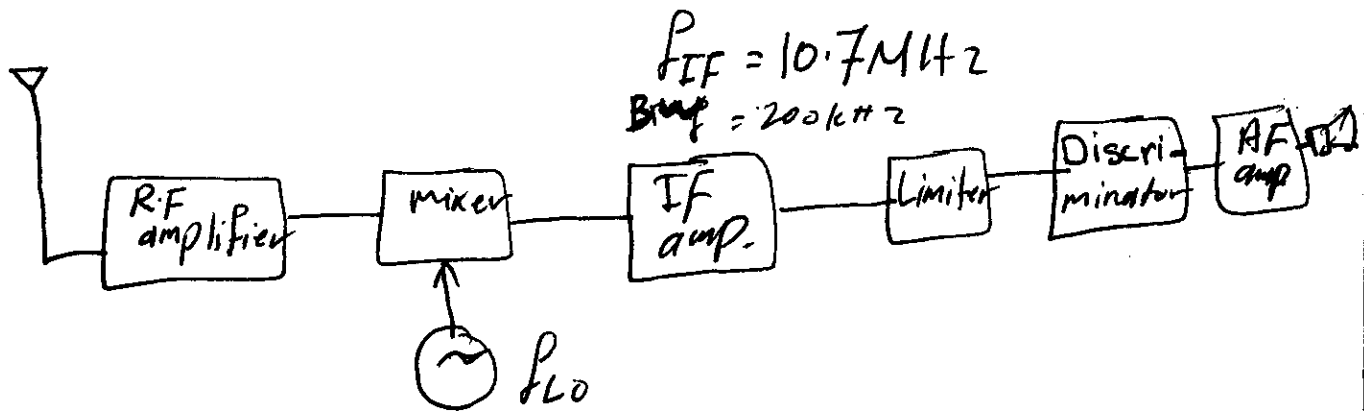
$$P(t) = A_c \omega_c [1 + \frac{K_f}{\omega_c} f(t)]$$

$$P_3(t) = A_c w_c + A_c k_f f(t)$$

by using D.C Blocking to Remove $A_c w_c$

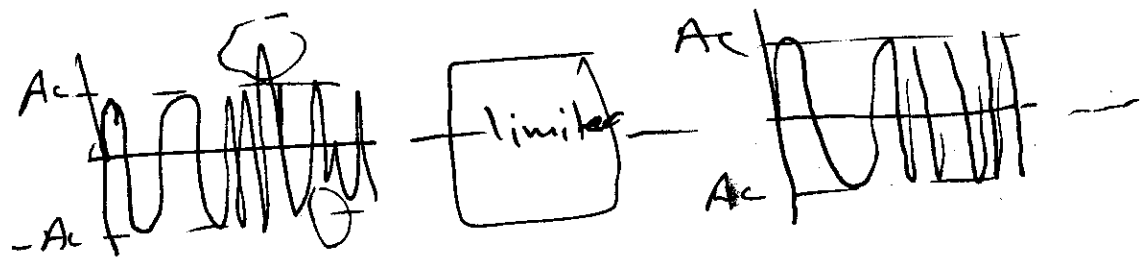
$$P_3(t) = A_c k_f f(t)$$

FM Receiver



- I.F Frequency In FM (10.7 MHz)

- Limiter; used to Remove any noise In Amplitude and fixed it at A_c



F.M Receiver have high Quality ~~and~~ Signal
~~for that used~~
 Transmitted by using UHF 88-108 MHz

Frequency Division Multiplexing (FDM)

It's possible to send several signals simultaneously by choosing a different carrier frequency for each signal.

These carrier frequencies are chosen so that the signal spectra are not overlapping or overlapping.

There are two type of carrier

- ① main carrier (main modulation)
- ② sub carrier (sub modulation)

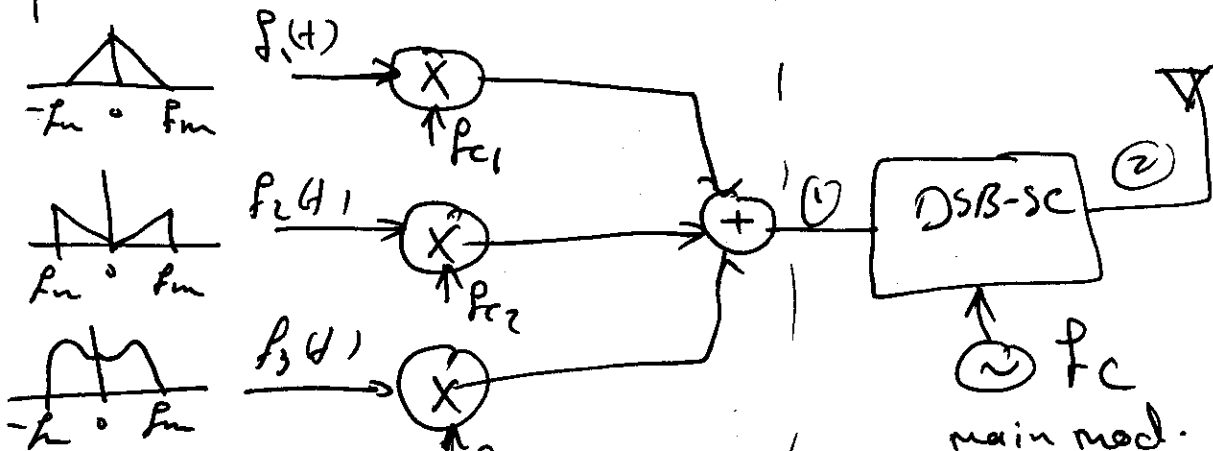
Ex Total Bandwidth = $n (BW_{\text{signal}} + f_g)$

n : no. of channel (FDM)

BW_{signal} : sub modulation Bandwidth

f_g : guard frequency

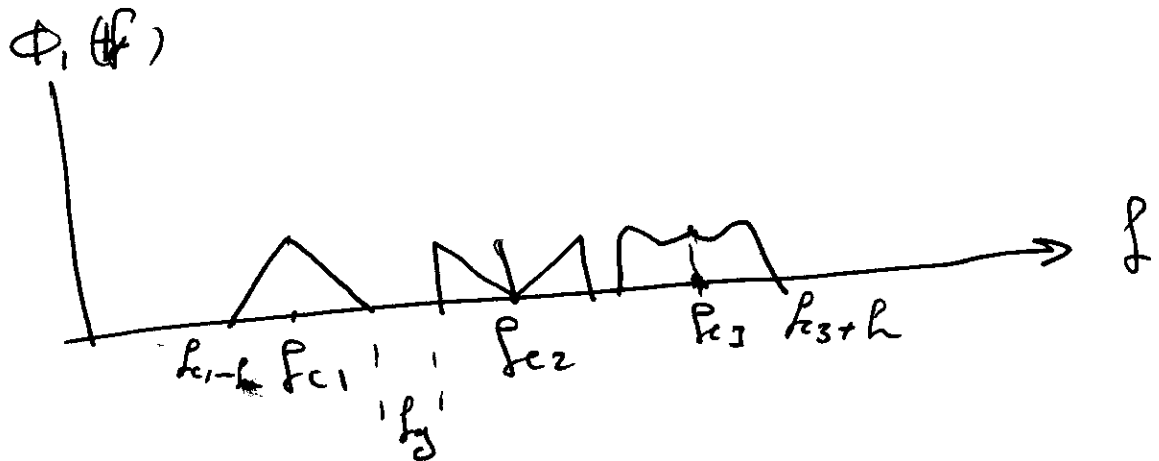
Exp/ used Three signal shown to draw output spectrum



sub carrier frequency

f_{c1}, f_{c2}, f_{c3}

no. of subcarrier = 3



$$BW_{T \text{ sub.}} = 3(2f_m + f_g)$$

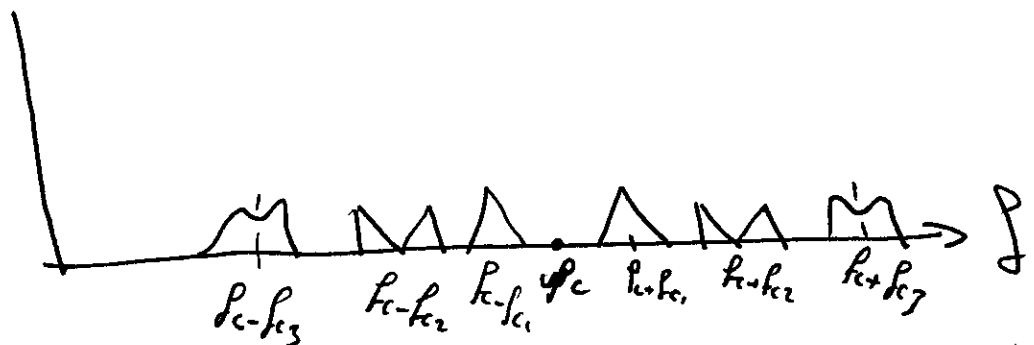
when signal

$$\Phi_1(t) = f_1(t) \cos \omega_{c1} t + f_2(t) \cos \omega_{c2} t + f_3(t) \cos \omega_{c3} t$$

after main modulation

$$\Phi_T(t) = \Phi_1(t) \cdot \cos \omega_c t$$

$$f_c > f_{c3} > f_{c2}$$



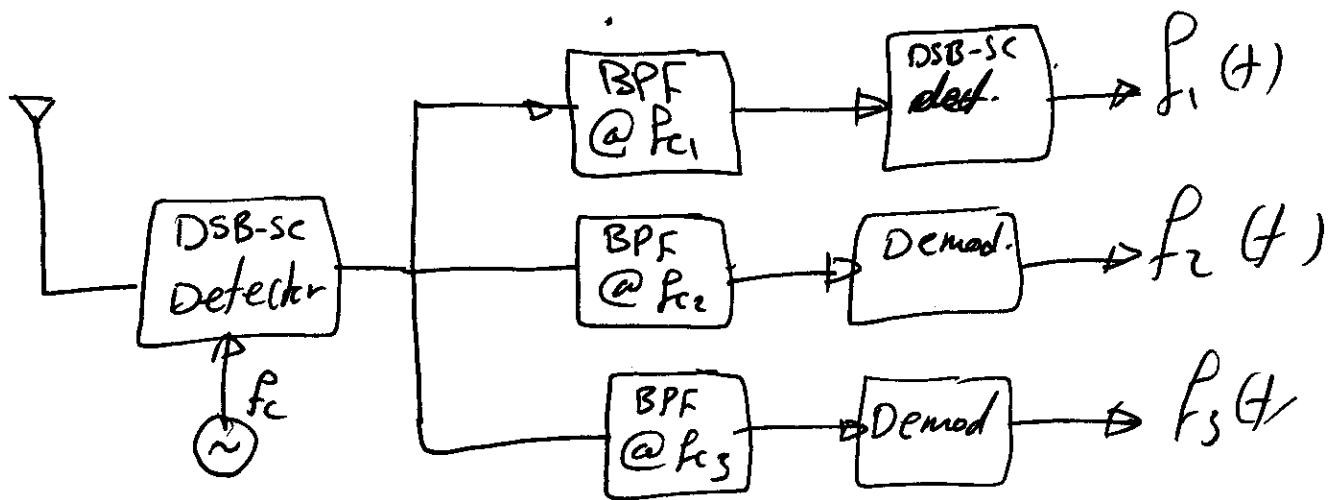
$$BW_T = 2 BW_1$$

$$= 6(2f_m + f_g)$$

FDM Receiver

To Received FDM used MULT-stage

- ① used BPF with different carrier frequency
- ② Detected In main modulation
- ③ Detect sub modulation



by using BPF with different center frequency
can't separated every multiplexing system.

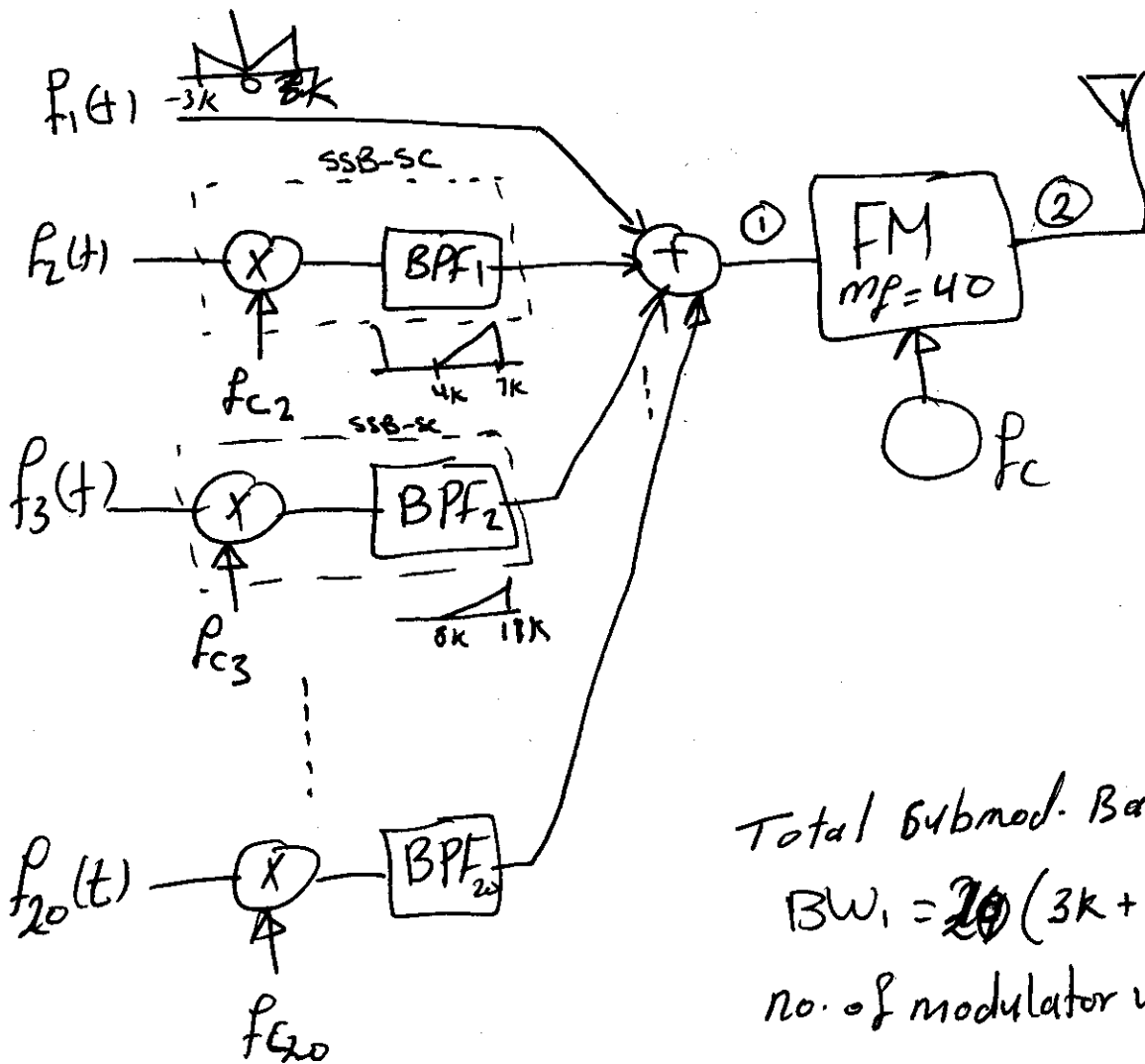
Hint

- Main & sub modulation can used any type of analogue or Digital Modulation Like (AM, DSB-SC, SSB-SC & FM)
- In Exchange used SSB-SC as main submodulator and FM as main modulated.

Exp/ Design FDM system to Multiplexed 20 channel used SSB-SC (USB) as sub carrier and FM of $m_f = 40$ as main modulation.
 (Hint: signal low pass signal of $f_m = 3\text{kHz}$ & guard frequency $f_g = 1\text{kHz}$, $f_{c1} = 0\text{Hz}$)

Sol.

① Design sub modulation to mod. SSB-SC

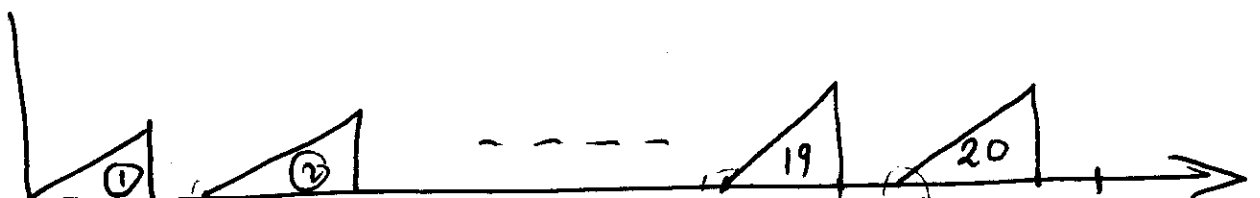


Total submod. Bandwidth

$$BW_1 = 20(3\text{K} + 1\text{K}) = 80\text{KHz}$$

$$\text{no. of modulator used} = \frac{n-1}{20-1} = 19$$

$$\text{no. of carrier used} = 19$$



From spectral

$$f_{c2} = 4 \text{ KHz}$$

$$f_{c7} = 24 \text{ KHz}$$

$$f_{c20} = 76 \text{ KHz}$$

$$f_{c3} = 8 \text{ KHz}$$

$$f_{c8} = 28 \text{ KHz}$$

$$f_{c4} = 12 \text{ KHz}$$

$$f_{c9} = 32 \text{ KHz}$$

$$f_{c5} = 16 \text{ KHz}$$

$$f_{c6} = 20 \text{ KHz}$$

Design BPF for modulator

BPF₁

$$BW_2 = 3 \text{ KHz}$$

$$f_{\text{cent}_2} = 5.5 \text{ KHz}$$



BPF₂

$$BW_3 = 3 \text{ KHz}$$

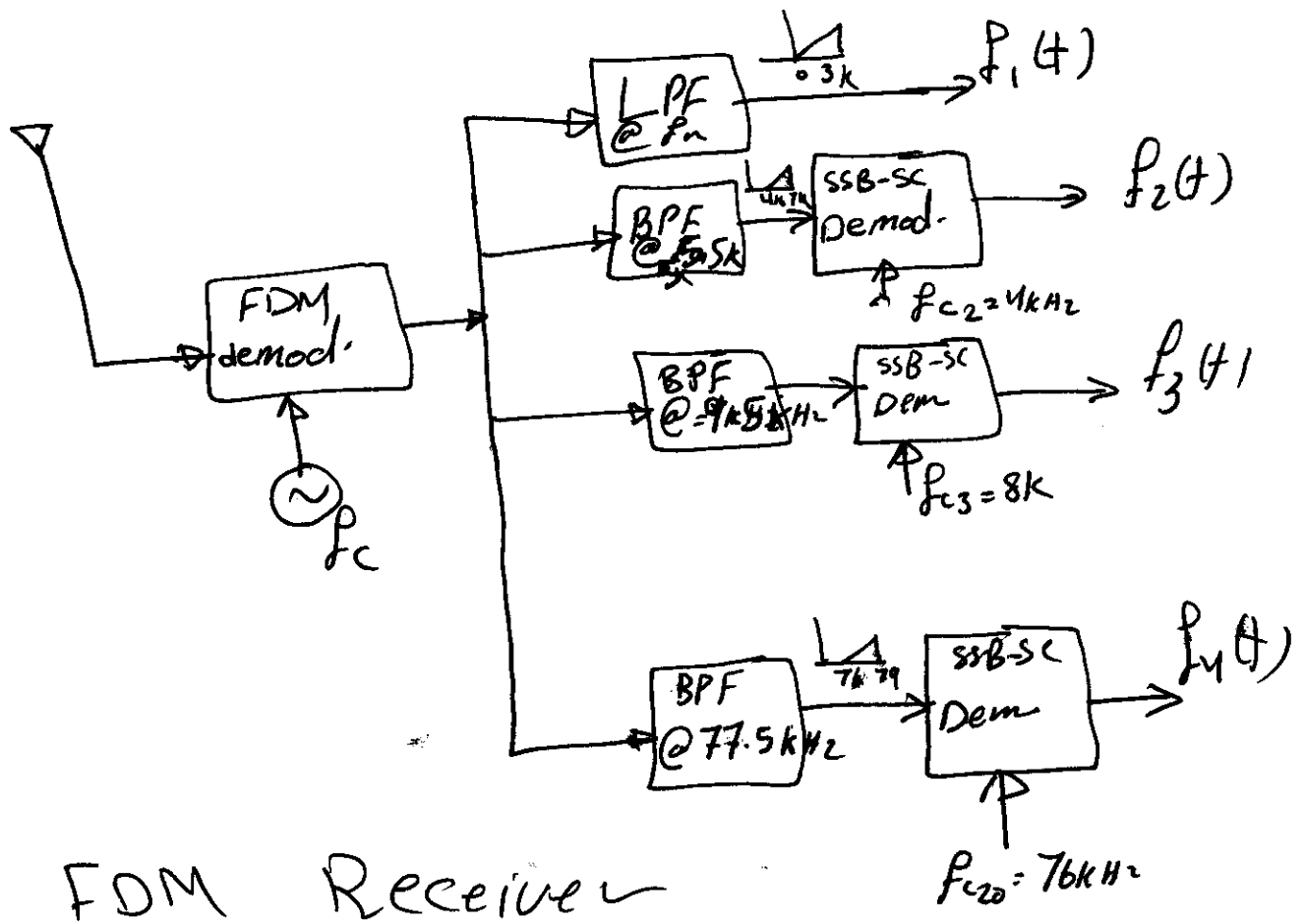
$$f_{\text{cent}_2} = 9.5 \text{ KHz}$$

Total Bandwidth

$$\begin{aligned} BW_{\text{FM}} &= 2 BW_1 (M_f + 1) \\ &= 2 \cdot 80 \text{ KHz} (4 + 1) = \end{aligned}$$

De

Design FDM Receiver



FDM Receiver

Q/ An FDM system have 100 channel In exchange use used (SSB-SC) (USB) as submodulator & DSB-SC as main modulator of carrier $f_c = 4\text{MHz}$ (guard freq. if channel have Low Pass signal $f_m = 4\text{kHz}$ & guard freq. $f_g = 1\text{kHz}$). Draw spectrum at every part of T & R?